

## SUBMARINE TATICAL GEOMETRIES DURING ENEMY VEHICLE ATTACK USING NOVEL STATISTICAL STOCHASTIC NONLINEAR FILTER

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Received: 19 September 2016, Revised and Accepted: 21 September 2016

### ABSTRACT

A particle filter (PF) is proposed for tracking a torpedo using bearings-only measurements when torpedo is attacking an ownship. Towed array is used to generate torpedo bearing measurements. Ownship evasive maneuver is used for observability of the bearings-only process. PF combined with modified gain bearings-only extended Kalman filter is used to estimate torpedo motion parameters, which are used to calculate optimum ownship evasive maneuver. Monte-Carlo simulation is carried out, and the results are presented for typical scenarios.

**Keywords:** Estimation, Evasive maneuver, Ownship, Particle filter, Simulation, Torpedo, Towed array.

### INTRODUCTION

In the sea environment, two-dimensional headings only target movement examination is for the most part utilized. An ownship screens loud sonar orientation from a transmitting target and discovers target movement parameters (TMP) - viz., range, course, bearing, and speed of the objective. The fundamental suppositions are that the objective moves at consistent speed more often than not. The ownship movement is unlimited. The objective and ownship are thought to be in the same flat plane. The issue is intrinsically nonlinear as the estimation is nonlinear. Direction bearing only tracking (BOT) is the assurance of the direction of an objective exclusively from bearing estimations. In this aloof target following, a solitary ownship screens a grouping of bearing estimations, which are thought to be accessible at equi-dispersed discrete times. The objective movement investigation can be seen as target confinement and its following. The BOT zone has been generally explored [1-4], and various answers for this issue have been proposed.

Since bearing estimations are extricated from single detached sonar, the procedure stays inconspicuous until ownship executes a legitimate move. For introducing the ideas in clear, it is accepted that the objective is moving at steady speed. Established minimum squares strategy and Kalman channel cannot be straightforwardly connected. One valuable methodology is the pseudo-linear estimator (PLE) detailing proposed in [1] which bumps the nonlinearities into the commotion term, bringing about a direct estimation condition.

Here, the estimation grid contains components that are elements of uproarious orientation and, by and large, are associated with the clamor terms of the estimation condition. Accordingly, the PLE displays an inclination in the evaluated TMP [1,5,6]. As it offers a non-veering arrangement, commonly PLE is utilized as a move down arrangement alongside the advanced sifting systems (which will be talked about in the blink of an eye). The established PLE which is as clump preparing is changed over into consecutive handling [7] so as to not require introduction of target state vector and this component is particularly valuable for sea submerged target following.

Greatest maximum likelihood estimator (MLE) is observed to be an appropriate calculation for inactive target following applications, by righteousness of its qualities [1]. This is angle look taking into account a group preparing of all the accessible estimations. MLE is asymptotically proficient, reliable, fair and its covariance grid approaches the Cramer - Rao destined for substantial specimens. Rather than accepting

some discretionary qualities, PLE yields are utilized for the introduction of MLE [8].

Another methodology, use of extended Kalman filter (EKF) in adjusted modified polar (MP) organizes [9] edge is observed to be helpful for this nonlinear application. In this calculation, the perceptible and inconspicuous segments of the evaluated state vector are consequently sick molding, which is the essential driver of shakiness. The MP state evaluations are asymptotically fair. A half and half arrange framework approach created by Walter Grossman is likewise another effective commitment to direction just latent target following [10].

Another effective commitment to this field is by Song and Speyer [11]. The difference in EKF [3,4] is dispensed with by altering the ownship picks up. This calculation is named as altered addition heading just developed Kalman channel (modified gain bearings-only EKF [MGBEKF]). The crucial thought behind MGBEKF is that the nonlinearities "modifiable." This calculation has a few likenesses with the pseudo estimation work, however, it is not the same. In pseudo estimation channel, the addition is an element of at various times estimations. It is to be noticed that MGBEKF depends on the calculation for the EKF, the increase of the MGBEKF is a component of just past estimations. Along these lines, by dispensing with the immediate relationship of the addition and estimation commotion process in the assessments of MGBEKF, the predisposition in the estimation is wiped out. An improved adaptation of the adjusted addition capacity is made accessible by Galkowski and Islam [12]. This adaptation is helpful for air applications, where height and bearing estimations are accessible. It is further adjusted for submerged target following applications [13,14], where direction just estimations are accessible.

The conventional Kalman channel is ideal when the model is straight. Tragically, a large portion of the state estimation issues like following of the objective utilizing course just data are nonlinear, in this manner restricting the reasonable convenience of the Kalman channel and EKF. Consequently, the attainability of a novel change, known as unscented change, which is intended to spread data as mean vector and covariance framework through a nonlinear procedure, is investigated for submerged applications. The unscented change is combined with specific parts of the great Kalman channel. It is less demanding to actualize and utilizes the same request of computations. Utilizing course just estimations, unscented Kalman filter (UKF) channel calculation gauges TMP [15,16]. UKF can be dealt with as a contrasting option to

MGBEKF. Yet at the same time, the fundamental imperative that is the pdf of commotion in the estimations is to be Gaussian, for ideal results. Consequently, UKF can take up nonlinearity however not non-Gaussian commotion in the estimations.

Particle filters (PF) [17-19] are the new era of cutting edge channels, which are helpful for nonlinear and non-Gaussian applications. PF or sequential Monte-Carlo strategies utilize an arrangement of weighted state tests, called particles, to estimated the back likelihood dissemination in a Bayesian setup. Anytime of time, the arrangement of particles can be utilized to estimated the pdf of the state. As the quantity of particles increment to limitlessness, the estimation approaches the genuine pdf. They give almost ideal state gauges on account of nonlinear and non-Gaussian frameworks, dissimilar to Kalman channel based methodologies. Since PFs do not inexact nonlinearities or non-Gaussian commotion in the framework and utilize an extensive number of particles, they have a tendency to be computationally unpredictable. Be that as it may, with the as of now accessible propelled microchips, the calculation can be effortlessly overseen. PF joined with MGBEKF (PFMGBEKF) is proposed in this paper for inactive heading just torpedo following utilizing towed cluster estimations.

The undertaking is to appraise the torpedo movement parameters, while ownship is in assault by a torpedo. Subsequent to getting the principal contact of the torpedo, ownship tries to escape by doing a specific move. This move depends on 700 relative bearing strategy, which is being utilized by Navy. Here, this first move is called as ownship wellbeing move. The thought is to escape from the field as right on time and fast as could be allowed. When all is said in done, the ownship tries to build the rate in the wake of swinging to the required course. This is required for the ownship to escape from the objective as ahead of schedule as would be prudent.

The ownship's consequent getaway moves can be done in efficient way, if torpedo's extent, bearing, course, and speed are known. As these are not accessible, these are assessed utilizing PFMGBEKF. Here as a course are just accessible, ownship well-being move will be utilized for discernibleness of the procedure. Amid wellbeing move, ownship tries to escape in a manner that extent among ownship and target gets to be most extreme worth with increment in time. Be that as it may, for getting arrangement, it is another route round. Reach ought to abatement to get additionally bearing rate with increment in time. With this limitation, ownship tries to assess the torpedo movement parameters to ascertain legitimate hesitant moves utilizing closest path of approach (CPA) at different time moments and break from torpedo assault.

Section 2 describes mathematical modeling of measurements, PFMGBEKF and CPA. PFMGBEKF is developed and implemented on PC platform using MATLAB. Section 3 describes about implementation aspects of the algorithm. Extensive simulation is carried out and the results are presented for three scenarios. Section 4 covers the limitations of the algorithm, and finally the paper is concluded in Section 5.

**MATHEMATICAL MODELING**

**State and measurement equations**

Let the target state vector be  $X_s(k)$  where,

$$X_s(k) = [\dot{x}(k) \ \dot{y}(k) \ R_x(k) \ R_y(k)]^T \tag{1}$$

Where  $\dot{x}(k)$  and  $\dot{y}(k)$  are target velocity components and,  $R_x(k)$  and  $R_y(k)$  are range components, respectively. The target state dynamic equation is given by,

$$X_s(k+1) = \phi X_s(k) + b(k+1) + \Gamma \omega(k) \tag{2}$$

Where  $\phi$  and  $b$  are transition matrix and deterministic vector, respectively. The transition matrix is given by,

$$\phi = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ t & 0 & 1 & 0 \\ 0 & t & 0 & 1 \end{bmatrix} \tag{3}$$

Where  $t$  is sample time,

$$b(k+1) = [0 \ 0 -\{x_o(k+1) - x_o(k)\} -\{y_o(k+1) - y_o(k)\}] \tag{4}$$

$$\Gamma = \begin{bmatrix} t & 0 & t^2/2 & 0 \\ 0 & t & 0 & t^2/2 \end{bmatrix} \tag{5}$$

Where  $x_o(k)$  and  $y_o(k)$  are ownship position components. The plant noise  $\omega(k)$  is assumed to be zero mean white Gaussian with covariance.

$$E[\omega(k)\omega'(k)] = Q\delta_{kj} \tag{6}$$

Where  $Q = \begin{bmatrix} t^2 & 0 & t^3/2 & 0 \\ 0 & t^2 & 0 & t^3/2 \\ t^3/2 & 0 & t^4/8 & 0 \\ 0 & t^3/2 & 0 & t^4/8 \end{bmatrix}$  tag(7)

True north convention is followed for all angles to reduce mathematical complexity and for easy implementation. The bearing measurement,  $B_m$  is modeled as,

$$B_m(k+1) = \tan^{-1} \left( \frac{r_x(k+1)}{r_y(k+1)} \right) + \zeta(k) \tag{8}$$

Where  $\zeta(k)$  is error in the measurement, and this error is assumed to be zero mean Gaussian with variance  $\sigma^2$ . The measurement and plant noises are assumed to be uncorrelated to each other. Equation 8 is a nonlinear equation and is linearized using the first term of the Taylor series for  $R_x$  and  $R_y$ . The measurement matrix is obtained as,

$$H(k+1) = \begin{bmatrix} 0 & 0 & \bar{R}_y(k+1/k)/\bar{R}^2(k+1/k) & -\bar{R}_x(k+1/k)/\bar{R}^2(k+1/k) \end{bmatrix} \tag{9}$$

Since the true values are not known, the estimated values of  $R_x$  and  $R_y$  are used in Equation 9.

**PF**

The PF is a statistical brute-force approach to estimation that often works well for problems (i.e., systems that are highly nonlinear) that are difficult for the conventional Kalman filter. Let us derive the basic idea of the PF, it was invented to numerically implement the Bayesian estimator. The main idea is intuitive and straight forward. At the beginning of the estimation problem, we randomly generate  $N$  state vectors based on the initial pdf  $P(X_s(0))$  (which is assumed to be known). These state vectors are called particles and are denoted as  $X_s(k/k)$  ( $k=1, 2, \dots, N$ ). At each time step, we propagate the particles to the next time step using the process equation.

$$X_s(k+1/k) = f(X_s(k-1/k), w(k+1)), (k=1, 2, \dots, N) \tag{10}$$

Where each  $w(k+1)$  noise vector is randomly generated on the basis of the known pdf of  $w(k)$ . After we receive the measurement at time  $k$ , we compute the conditional relative likelihood of each particle  $X_s(k+1/k)$ . That is, we evaluate the pdf  $P(Z(k)|X_s(k+1/k))$ . This can be done if we know the nonlinear measurement equation and the pdf of the measurement noise. For example, if an  $m$ -dimensional measurement equation is given as  $Z(k) = h(X_s(k)) + v(k)$  and  $v(k) \sim N(0, R)$  then a relative likelihood  $q(k)$ , that the measurement is equal to a specific measurement

$z^*$  given the premise that  $X_s(k)$  is equal to the particle  $X_s(k+1/k)$  can be computed as follows [18].

$$q(k) = P[Z(k) = z^* | X_s(k) = X_s(k+1/k)]$$

$$= P[v(k) = z^* - h(X_s(k+1/k))]$$

$$\sim \frac{1}{(2\pi)^m/2 |R|^{1/2}} \exp\left(-\frac{[z^* - h(X_s(k+1/k))]^T R^{-1} [z^* - h(X_s(k+1/k))]}{2}\right)$$
(11)

The  $\sim$  symbol in the above equation means that the probability is not really given by the expression on the right side, but the probability is directly proportional to the right side. Hence, if this equation is used for all the particles,  $X_s(k+1/k)$  ( $k=1, 2, \dots, N$ ), then the relative likelihoods that the state is equal to each particle will be correct. Now we normalize the relative likelihoods obtained in Equation 11 as follows.

$$q(k) = \frac{q(k)}{\sum_{i=1}^N q(i)}$$
(12)

Now we resample the particles from the computed likelihoods and a new set of particles that are randomly generated on the basis of the relative likelihoods  $q(k)$ .

**Particle filtering combined with other filters**

One approach that has been proposed for improving particle filtering is to combine it with another filter such as the EKF, UKF, or MGBEKF [18]. In this approach, each particle is updated at the measurement time using the EKF, UKF or MGBEKF and then resampling (if required) is performed using the measurement. This is like running a bank of  $N$  Kalman filters (one for each particle) and then adding a resampling step after each measurement. After  $X_s(k+1/k)$  is obtained, it can be refined using the EKF, UKF or MGBEKF measurement-update equations. In this paper, PF is combined with the MGBEKF.  $X_s(k+1/k)$  is updated to  $X_s(k+1/k+1)$  according to the following MGBEKF equations [18].

$$P(k+1/k)_i = \Phi(k+1/k)_i P(k/k)_i \Phi^T(k+1/k)_i + \Gamma Q(k+1) \Gamma^T$$
(13)

$$G(k+1)_i = P(k+1/k)_i H^T(k+1)_i [\sigma^2 + H(k+1)_i P(k+1/k)_i H^T(k+1)_i]^{-1}$$
(14)

$$X_s(k+1/k+1)_i = X_s(k+1/k)_i + G(k+1)_i [B_m(k+1)_i - h(k+1, X_s(k+1/k)_i)]$$
(15)

$$P(k+1/k+1)_i = [I - G(k+1)_i g(B_m(k+1)_i, X_s(k+1/k)_i)]$$

$$\cdot P(k+1/k)_i [I - G(k+1)_i g(B_m(k+1)_i, X_s(k+1/k)_i)]^T$$

$$+ \sigma^2 G(k+1)_i G^T(k+1)_i$$
(16)

Where  $G(k+1)$  is Kalman gain,  $P(k+1/k)$  is a priori estimation error covariance for the  $i^{th}$  particle and  $g(\cdot)$  is modified gain function.  $g(\cdot)$  is given by,

$$g = \begin{bmatrix} 0 & 0 & \cos B_m / (\hat{R}_x \sin B_m + \hat{R}_y \cos B_m) & -\sin B_m / (\hat{R}_x \sin B_m + \hat{R}_y \cos B_m) \end{bmatrix}$$
(17)

Since true bearing is not available in practice, it is replaced by the measured bearing to compute the function  $g(\cdot)$ .

**Resampling**

In every update of PFMGBEKF, it is monitored to decide whether resampling of particles in respect of target state vector and its

covariance matrix is required or not. Resampling is required when the effective sample size,  $N_{eff} < N/3$  [18].

Where  $N_{eff} = \frac{1}{\sum_{i=1}^N q_i^2}$

(18)

Whenever resampling is required, the following procedure based on weights of particles is adopted. In this method, weights are sorted in descending order. The corresponding original indexes before sorting are remembered. Then, replication of particles (both the state and covariance matrices) is carried out in proportion to the weights of the particles starting with the particle with maximum weightage. This procedure is repeated for the particle with the next maximum weightage. This process is continued till all the particle positions are filled up. This method is close to the method suggested by Ristic *et al.* [17].

**CPA**

Let us assume that a target and ownship are moving at predefined constant velocities. At a certain point of time, these vehicles move through a point at which minimum distance will be there between them. This minimum distance is called CPA. Once torpedo motion parameters are estimated using PFMGBEKF, CPAs are calculated for all possible ownship evasive courses (say 0-360 in step of 1°). Ownship will do evasive maneuver in the course at which maximum CPA is generated. CPA is calculated as follows.

It is assumed that target motion parameters and ownship parameters are known. Initially, ownship is at the origin. Let the ownship and target courses be  $\phi$  and  $\psi$ , respectively. The distance between target and ownship positions at time  $t$  can be derived as follows (Fig. 1):

$$x_t = R \sin B + (V_t \sin \psi - V_o \sin \phi) t$$
(19)

$$y_t = R \cos B + (V_t \cos \psi - V_o \cos \phi) t$$
(20)

Where  $V_t$  and  $V_o$  are the speeds of target and ownship, respectively.

To simplify the Equation 20.

Let  $p = R \sin B$

$q = R \cos B$

$m = (V_t \sin \psi - V_o \sin \phi)$

$n = (V_t \cos \psi - V_o \cos \phi)$

Then eqn.(19) & eqn. (20) become,

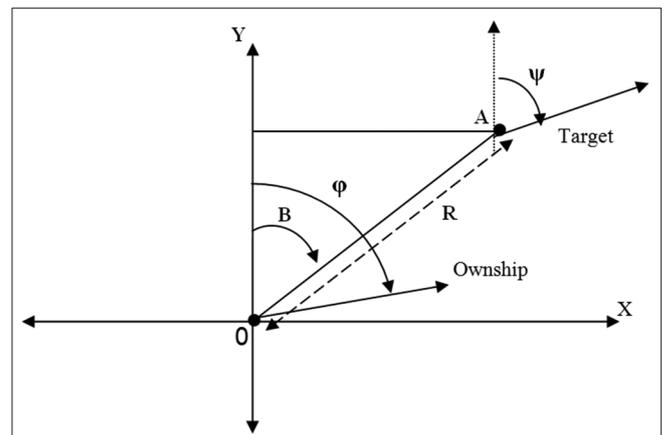


Fig. 1: Ownship and target encounter

$$x_t = (p + mt) \tag{21}$$

$$y_t = (q + nt) \tag{22}$$

The distance  $R_t$  between ownship and target is given by

$$R_t = \sqrt{(p + mt)^2 + (q + nt)^2}$$

By differentiating  $R_t^2$  w. r. t to time and equating it to zero,

$$\frac{d}{dt} (R_t^2) = 2(m^2 + n^2)t + 2(m p + n q) = 0 \tag{23}$$

For a particular value of  $t$  say  $t = t_m$  Equation (23) can be written as

$$t_m = \frac{(pm + qn)}{m^2 + n^2} \tag{24}$$

At this stage, taking second derivative, we have,

$$\frac{d^2}{dt^2} (R_t^2) = 2(m^2 + n^2) \tag{25}$$

And it is always  $>0$ . Hence,  $t_m$  gives minimum time at which the distance  $R$  is minimum. If  $t_m \leq 0$ , it implies that present range is CPA and time to reach CPA point is zero. If  $t_m > 0$ , substituting the value of  $t_m$  in Equation 21, we will get  $R_t^2$  as follows:

$$\begin{aligned} R_t^2 &= (p^2 + q^2) + (m^2 + n^2) \left( -\frac{(pm + qn)}{(m^2 + n^2)} \right)^2 + 2(pm + qn) \left( -\frac{(pm + qn)}{(m^2 + n^2)} \right) \\ &= (p^2 + q^2) + (pm + qn)^2 / (m^2 + n^2) - 2(pm + qn)^2 / (m^2 + n^2) \\ &= (p^2 + q^2) - (pm + qn)^2 / (m^2 + n^2) \end{aligned} \tag{26}$$

since  $R^2 = p^2 + q^2$ , Equation 26 can be modified as follows:

$$R_t^2 = R^2 - (p^*m + q^*n)^2 / (m^2 + n^2) \tag{27}$$

$R_t$  is nothing but CPA. So,

$$CPA = \sqrt{\left[ \frac{R^2 - (pm + qn)^2}{m^2 + n^2} \right]} \tag{28}$$

**IMPLEMENTATION AND SIMULATION**

For the implementation of the algorithm, the initial estimate of target state vector is chosen as follows. As only bearing measurements are available, it is not possible to guess the velocity components of the target. Hence, these components are each assumed as 15 m/second, which are close to the realistic speed of the torpedo. The range of the day, say 10,000 m, can be utilized in the calculation of initial position components of the torpedo as follows:

$$X(0|0) = [15 \ 15 \ 10000 \ \sin B_m \ 10000 \ \cos B_m]^T \tag{29}$$

It is assumed that the initial estimate,  $X(0|0)$  is uniformly distributed. Then, the elements of initial covariance diagonal matrix can be written as,

$$P(0|0) = \text{Diag} \left[ \frac{4 * x^2(0/0)}{12} \quad \frac{4 * y^2(0/0)}{12} \quad \frac{4 * r_x^2(0/0)}{12} \quad \frac{4 * r_y^2(0/0)}{12} \right] \tag{30}$$

As PF is combined with MGBEKF, 1000 particles (almost similar performance is achieved with 10000 particles) are used to estimate target motion parameters.

The measurement interval is assumed to be 1 second. It is also assumed that TA maximum auto detection range limit is 10,000 m. Estimation of torpedo motion parameters is stopped when the range is 500 m. Maximum ownship speed is 11 m/second. Ownship turning rate is considered 1°/second. It is assumed that measurements are corrupted with 1° r.m.s error of Gaussian distribution. All angles are considered with respect to True North 0-360°, clockwise positive. For the purpose of presentation, three scenarios as shown in Table 1 are considered for evaluation of the algorithm. The results obtained for the scenarios 1-3 are shown in Figs. 2-4, respectively. The estimated solution is said to be converged when,

- a. Error in the range estimate  $\leq 20\%$  of the actual range
- b. Error in the course estimate  $\leq 5^\circ$
- c. Error in the speed estimate  $\leq 4$  knots.

The convergence time to obtain all the target motion parameters with the required accuracy for each scenario is shown in Table 1. The

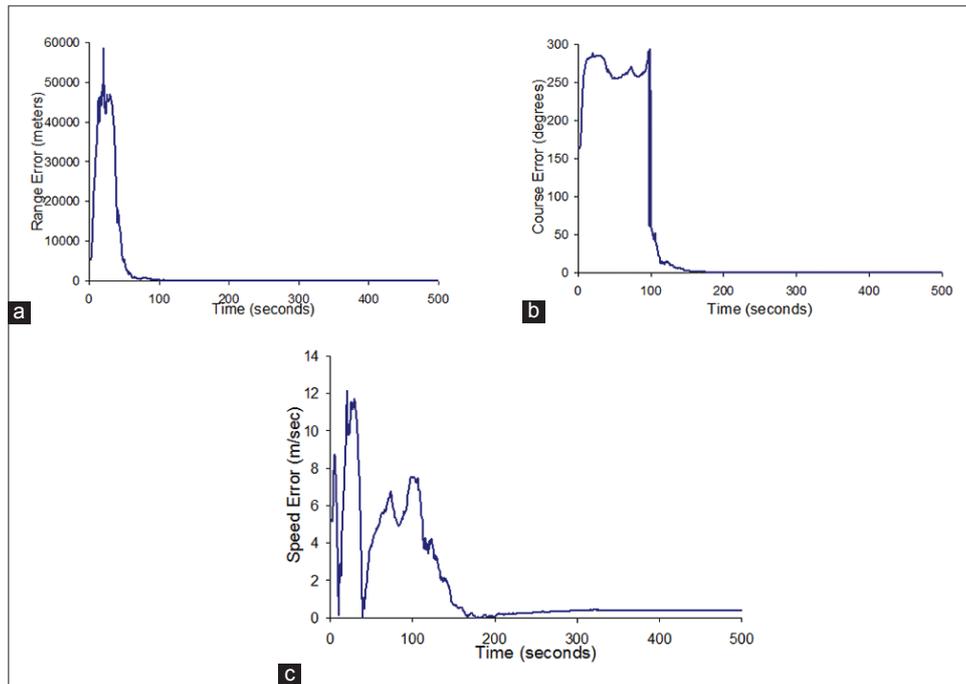


Fig. 2: (a) Error in range estimate, (b) error in course estimate, (c) error in speed estimate for scenario 1

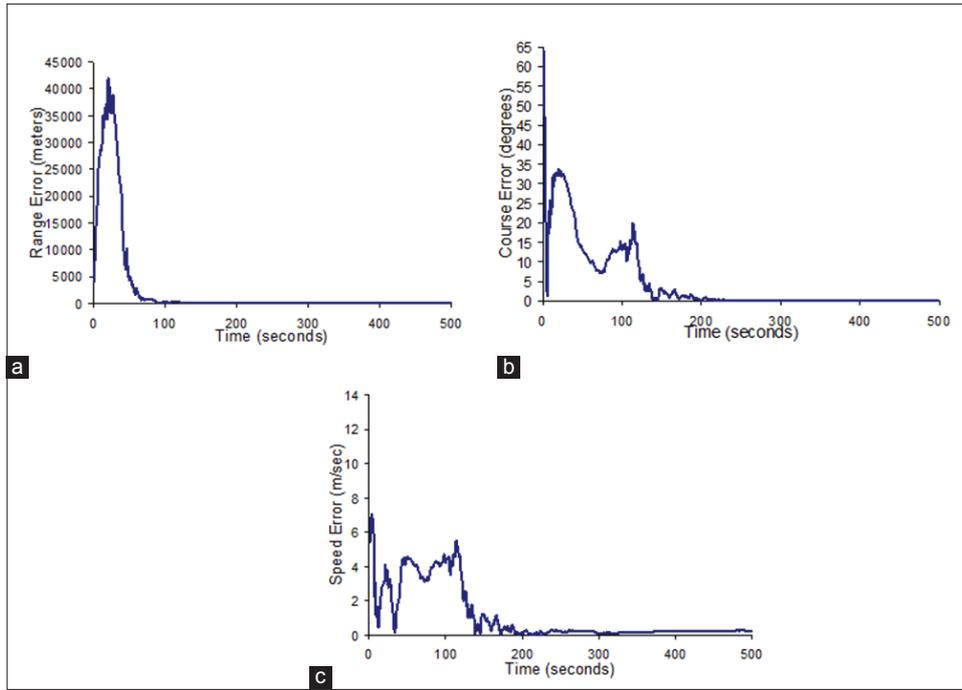


Fig. 3: (a) Error in range estimate, (b) error in course estimate, (c) error in speed estimate for scenario 2

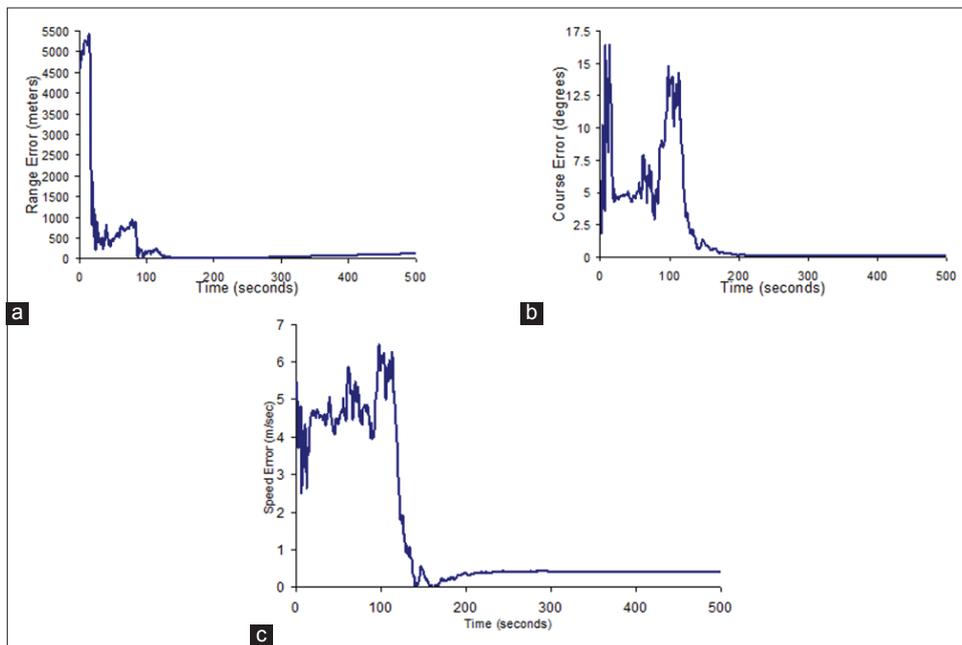


Fig. 4: (a) Error in range estimate, (b) error in course estimate, (c) error in speed estimate for scenario 3

Table 1: Geometrical scenarios

S. No.	Initial range (m)	Initial bearing (°)	Target speed (m/second)	Target course (°)	Ownship speed (m/second)	Ownship course (°)	Convergence time (second)
1	4500	90	15.45	293 (0°)	6.18	0°	145
2	6000	270	15.45	66.42 (0°)	6.18	0°	128
3	5000	320	15.45	125 (0°)	6.18	0°	124

ownship evasive maneuver for each scenario is based on CPA. As it is straightforward to find out maximum CPA using Equation 28, CPA results are not presented in the paper.

**Limitations of the algorithm**

Angle on target bow (ATB) is the angle between the target course and line of sight. When ATB is more than 60°, the distance between the

target and ownship increases as time increases and the bearing rate decreases substantially with the increase in a number of samples. In such situation, it is very difficult to track the target. Furthermore, the algorithm cannot provide good results when the measurement noise is more than  $1^\circ$  r.m.s. In general, these two situations are constraints to any type of filtering technique.

### CONCLUSION

PF (which is useful for nonlinear and non-Gaussian applications) combined with MGBEKF is proposed to estimate target motion parameters in passive target tracking. The performance of the PFMGBEKF is greatly superior to the standard EKF. In this paper, tracking of torpedo using towed array measurements is explored. Ownship safety maneuver is used for observability of the process. CPA method uses the estimated torpedo motion parameters to find out ownship evasive maneuver. Extensive simulation is carried out, and the results are found to be consistent. For the purpose of presentation, results of three typical scenarios are presented.

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