SUBMARINE TACTICAL GEOMETRIES DURING ENEMY VEHICLE ATTACK USING NOVEL STATISTICAL STOCHASTIC NONLINEAR FILTER

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ABSTRACT

A particle filter (PF) is proposed for tracking a torpedo using bearings-only measurements when torpedo is attacking an ownship. Towed array is used to generate torpedo bearing measurements. Ownship evasive maneuver is used for observability of the bearings-only process. PF combined with modified gain bearings-only extended Kalman filter is used to estimate torpedo motion parameters, which are used to calculate optimum ownship evasive maneuver. Monte-Carlo simulation is carried out, and the results are presented for typical scenarios.

Keywords: Estimation, Evasive maneuver, Ownship, Particle filter, Simulation, Torpedo, Towed array.

INTRODUCTION

In the sea environment, two-dimensional headings only target movement examination is for the most part utilized. An ownship screens load sonar orientation from a transmitting target and discovers target movement parameters (TMP) - viz., range, course, bearing, and speed of the objective. The fundamental suppositions are that the objective moves at consistent speed more often than not. The ownship movement is unlimited. The objective and ownship are thought to be in the same flat plane. The issue is intrinsically nonlinear as the estimation is nonlinear. Direction bearing only tracking (BOT) is the assurance of the direction of an objective exclusively from bearing estimations. In this abof target following, a solitary ownship screens a grouping of bearing estimations, which are thought to be accessible at equi-dispersed discrete times. The objective movement investigation can be seen as target confinement and its following. The BOT zone has been generally explored [1-4], and various answers for this issue have been proposed.

Since bearing estimations are extricated from single detached sonar, the procedure stays inconspicuous until ownship executes a legitimate move. For introducing the ideas in clear, it is accepted that the objective is moving at steady speed. Established minimum squares strategy and Kalman channel cannot be straightforwardly connected. One valuable methodology is the pseudo-linear estimator (PLE) detailing proposed in [1] which bumps the nonlinearities into the commotion term, bringing about a direct estimation condition.

Here, the estimation grid contains components that are elements of uprooted orientation and, by and large, are associated with the clamor or terms of the estimation condition. Accordingly, the PLE displays an inclinacion in the evaluated TMP [1,5,6]. As it offers a non-veering arrangement, commonly PLE is utilized as a move down arrangement alongside the advanced sifting systems (which will be talked about in the blink of an eye). The established PLE which is as clump preparing arrangement, commonly PLE is utilized as a move down arrangement alongside the advanced sifting systems (which will be talked about in the blink of an eye). The established PLE which is as clump preparing arrangement, commonly PLE is utilized as a move down arrangement alongside the advanced sifting systems (which will be talked about in the blink of an eye). 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At each time step, we propagate the particles to respectively. The transition matrix is given by,

$$
\Phi = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
t & 0 & 1 & 0 \\
0 & t & 0 & 1
\end{bmatrix}
$$

Where t is sample time,

$$
b(k+1) = \begin{bmatrix}
0 & 0 & -R_x(k+1) - x(k) \\
0 & 0 & -R_y(k+1) - y(k)
\end{bmatrix}
$$

The undertaking is to appraise the torpedo movement parameters, while ownership is in assault by a torpedo. Subsequent to getting the principal contact of the torpedo, ownership tries to escape by doing a specific move. This move depends on 700 relative bearing strategy, which is being utilized by Navy. Here, this first move is called as ownership wellbeing move. The thought is to escape from the field as right on time and fast as could be allowed. When all is said and done, the ownership tries to build the rate in the wake of swinging to the required course. This is required for the ownership to escape from the objective as ahead of schedule as would be prudent.

The ownership’s consequent getaway moves can be done in efficient way, if torpedo’s extent, bearing, course, and speed are known. As these are not accessible, these are assessed utilizing PF MGBEKF. Here as a course are just accessible, ownership well-being move will be utilized for discernibility of the procedure. Amid wellbeing move, ownership tries to escape in a manner that extent among ownership and target gets to be most extreme worth with increment in time. Be that as it may, for getting arrangement, it is another route round. Reach ought to abatement to get additionally bearing rate with increment in time. With this limitation, ownership tries to assess the torpedo movement parameters to ascertain legitimate hesitant moves utilizing closest path of approach (CPA) at different time moments and break from torpedo assault.

Section 2 describes mathematical modeling of measurements, PF MGBEKF and CPA. PF MGBEKF is developed and implemented on PC platform using MATLAB. Section 3 describes about implementation aspects of the algorithm. Extensive simulation is carried out and the results are presented for three scenarios. Section 4 covers the limitations of the algorithm, and finally the paper is concluded in Section 5.

**MATHEMATICAL MODELING**

**State and measurement equations**

Let the target state vector be \( X(k) \) where,

$$
X(k) = \begin{bmatrix}
x(k) \\
y(k) \\
R_x(k) \\
R_y(k)
\end{bmatrix}
$$

Where \( x(k) \) and \( y(k) \) are target velocity components and, \( R_x(k) \) and \( R_y(k) \) are range components, respectively. The target state dynamic equation is given by,

$$
X(k+1) = \Phi X(k) + B(k+1) + \omega(k)
$$

Where \( \Phi \) and \( B \) are transition matrix and deterministic vector, respectively. The transition matrix is given by,

$$
\Phi = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
t & 0 & 1 & 0 \\
0 & t & 0 & 1
\end{bmatrix}
$$

Where \( \omega(k) \) is assumed to be zero mean white Gaussian with covariance.

$$
E[\omega(k)\omega'(k)] = Q \delta_{kj}
$$

The measurement matrix is obtained as,

$$
H(k) = \begin{bmatrix}
t^2 & 0 & t^3/2 & 0 \\
0 & t^2 & 0 & t^3/2 \\
t^3/2 & 0 & t^4/8 & 0 \\
0 & t^3/2 & 0 & t^4/8
\end{bmatrix}
$$

True north convention is followed for all angles to reduce mathematical complexity and for easy implementation. The bearing measurement, \( B_m \) is modeled as,

$$
B_m(k+1) = \tan^{-1}\left(\frac{R_y(k+1)}{R_x(k+1)} + \zeta(k)\right)
$$

Where \( \zeta(k) \) is error in the measurement, and this error is assumed to be zero mean Gaussian with variance \( \sigma^2 \). The measurement and plant noises are assumed to be uncorrelated to each other. Equation 8 is a nonlinear equation and is linearized using the first term of the Taylor series for \( R_x \) and \( R_y \). The measurement matrix is obtained as,

$$
H(k+1) = \begin{bmatrix}
0 & 0 & R_y(k+1)/R_x(k+1) & -R_x(k+1)/R_x(k+1)
\end{bmatrix}
$$

Since the true values are not known, the estimated values of \( R_x \) and \( R_y \) are used in Equation 9.

**PF**

The PF is a statistical brute-force approach to estimation that often works well for problems (i.e., systems that are highly nonlinear) that are difficult for the conventional Kalman filter. Let us derive the basic idea of the PF, it was invented to numerically implement the Bayesian estimator. The main idea is intuitive and straightforward. At the beginning of the estimation problem, we randomly generate \( N \) state vectors based on the initial pdf \( P(X(0)) \) which is assumed to be known. These state vectors are called particles and are denoted as \( X(k) \) \( k = 1, 2, ..., N \). At each time step, we propagate the particles to the next time step using the process equation.

$$
X(k+1) = \{X(k+1 | k), w(k+1)\}, (k = 1, 2, ..., N)
$$

Where each \( w(k+1) \) noise vector is randomly generated on the basis of the known pdf of \( w(k) \). After we receive the measurement at time \( k \), we compute the conditional relative likelihood of each particle \( X(k+1 | k) \). That is, we evaluate the pdf \( P(Z(k) | X(k+1 / k)) \). This can be done if we know the nonlinear measurement equation and the pdf of the measurement noise. For example, if an m-dimensional measurement equation is given as \( Z(k) = h(X(k)) + v(k) \) and \( v(k) \sim N(0, R) \) then a relative likelihood \( q(k) \), that the measurement is equal to a specific measurement
After siqkT(18), is obtained, it can be refined using the EKF, UKF or MGBEKF. In every update of PFMGBEKF, it is monitored to decide whether resampling of particles in respect of target state vector and its covariance matrix is required or not. Resampling is required when the effective sample size, Neff < N/3 [18].

\[
N_{\text{eff}} = \frac{1}{\sum_{i=1}^{N} q_i^2}
\]

Whenever resampling is required, the following procedure based on weights of particles is adopted. In this method, weights are sorted in descending order. The corresponding original indexes before sorting are remembered. Then, replication of particles (both the state and covariance matrices) is carried out in proportion to the weights of the particles starting with the particle with maximum weightage. This procedure is repeated for the particle with the next maximum weightage. This process is continued till all the particle positions are filled up. This method is close to the method suggested by Ristick et al [17].

**CPA**

Let us assume that a target and ownership are moving at predefined constant velocities. At a certain point of time, these vehicles move through a point at which minimum distance will be there between them. This minimum distance is called CPA. Once torpedo motion parameters are estimated using PFMGBEKF, CPAs are calculated for all possible ownership evasive courses (say 0-360 in step of 1°). Ownership will do evasive maneuver in the course at which maximum CPA is generated. CPA is calculated as follows.

It is assumed that target motion parameters and ownership parameters are known. Initially, ownership is at the origin. Let the ownership and target courses be ϕ and ψ, respectively. The distance between target and ownership positions at time t can be derived as follows (Fig. 1):

\[
x = R \sin \beta + (V_t \sin \psi - V_o \sin \phi) t
\]

\[
y = R \cos \beta + (V_t \cos \psi - V_o \cos \phi) t
\]

Where Vt and Vo are the speeds of target and ownership, respectively. To simplify the Equation 20.

Let p = R sin β

q = R cos β

m = (V_t \sin \psi - V_o \sin \phi)

m = (V_t \cos \psi - V_o \cos \phi)

Then eqn. (19) & eqn. (20) become:

**Resampling**

In every update of PFMGBEKF, it is monitored to decide whether resampling of particles in respect of target state vector and its covariance is required or not. Resampling is required when the effective sample size, Neff < N/3 [18].

\[
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The distance \( R \) between ownship and target is given by

\[
R = \sqrt{(p + mt)^2 + (q + nt)^2}
\]

By differentiating \( R^2 \) w.r.t to time and equating it to zero,

\[
\frac{d}{dt} \left( R^2 \right) = 2(m \frac{p}{m^2 + n^2} + n \frac{q}{m^2 + n^2}) t + 2 (m p + n q) = 0
\]

For a particular value of \( t = t_m \) Equation (23) can be written as

\[
R^2 = \frac{(pm + qn)^2}{m^2 + n^2}
\]

At this stage, taking second derivative, we have,

\[
\frac{d^2}{dt^2} \left( R^2 \right) = 2(m^2 + n^2)
\]

And it is always >0. Hence, \( t_m \) gives minimum time at which the distance \( R \) is minimum. If \( t_m \leq 0 \), it implies that present range is CPA and time to reach CPA point is zero. If \( t_m > 0 \), substituting the value of \( t_m \) in Equation 21, we will get \( R^2 \) as follows:

\[
R^2 = \frac{(p^2 + q^2)}{2(m^2 + n^2)} \left( (p m + q n)^2 - (p m - q n)^2 \right)
\]

Where \( R = p^2 + q^2 \), Equation 26 can be modified as follows:

\[
R = \sqrt{R^2 - \frac{(pm + qn)^2}{m^2 + n^2}}
\]

CPA = \left( \frac{R^2 - (pm + qn)^2}{m^2 + n^2} \right)

**IMPLEMENTATION AND SIMULATION**

For the implementation of the algorithm, the initial estimate of target state vector is chosen as follows. As only bearing measurements are available, it is not possible to guess the velocity components of the target. Hence, these components are each assumed as 15 m/second, which are close to the realistic speed of the torpedo. The range of the day, say 10,000 m, can be utilized in the calculation of initial position components of the torpedo as follows:

\[
X(0|0) = \begin{bmatrix} 15 & 15 & 10000 \sin B_m & 10000 \cos B_m \end{bmatrix}^T
\]

It is assumed that the initial estimate, \( X(0|0) \) is uniformly distributed. Then, the elements of initial covariance diagonal matrix can be written as,

\[
P(0/0) = \text{Diag} \begin{bmatrix} 4* \hat{x}_r^2(0/0) & 12 & 4* \hat{y}_r^2(0/0) & 12 & 4* \hat{z}_r^2(0/0) & 12 \end{bmatrix}
\]

As PF is combined with MGBEKE, 1000 particles (almost similar performance is achieved with 10000 particles) are used to estimate target motion parameters.

The measurement interval is assumed to be 1 second. It is also assumed that TA maximum auto detection range limit is 10,000 m. Estimation of torpedo motion parameters is stopped when the range is 500 m. Maximum ownship speed is 11 m/second. Ownship turning rate is considered 1°/second. It is assumed that measurements are corrupted with 1° r.m.s error of Gaussian distribution. All angles are considered with respect to True North 0-360°, clockwise positive. For the purpose of presentation, three scenarios as shown in Table 1 are considered for evaluation of the algorithm. The results obtained for the scenarios 1-3 are shown in Figs. 2-4, respectively. The estimated solution is said to be converged when,

- a. Error in the range estimate ≤ 20% of the actual range
- b. Error in the course estimate ≤ 5°
- c. Error in the speed estimate ≤ 4 knots.

The convergence time to obtain all the target motion parameters with the required accuracy for each scenario is shown in Table 1.

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ownship evasive maneuver for each scenario is based on CPA. As it is straightforward to find out maximum CPA using Equation 28, CPA results are not presented in the paper.

Limitations of the algorithm
Angle on target bow (ATB) is the angle between the target course and line of sight. When ATB is more than 60°, the distance between the

Table 1: Geometrical scenarios

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Initial range (m)</th>
<th>Initial bearing (°)</th>
<th>Target speed (m/second)</th>
<th>Target course (°)</th>
<th>Ownship speed (m/second)</th>
<th>Ownship course (°)</th>
<th>Convergence time (second)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4500</td>
<td>90</td>
<td>15.45</td>
<td>293 (0°)</td>
<td>6.18</td>
<td>0°</td>
<td>145</td>
</tr>
<tr>
<td>2</td>
<td>6000</td>
<td>270</td>
<td>15.45</td>
<td>66.42 (0°)</td>
<td>6.18</td>
<td>0°</td>
<td>128</td>
</tr>
<tr>
<td>3</td>
<td>5000</td>
<td>320</td>
<td>15.45</td>
<td>125 (0°)</td>
<td>6.18</td>
<td>0°</td>
<td>124</td>
</tr>
</tbody>
</table>

Fig. 3: (a) Error in range estimate, (b) error in course estimate, (c) error in speed estimate for scenario 2

Fig. 4: (a) Error in range estimate, (b) error in course estimate, (c) error in speed estimate for scenario 3
target and ownship increases as time increases and the bearing rate decreases substantially with the increase in a number of samples. In such situation, it is very difficult to track the target. Furthermore, the algorithm cannot provide good results when the measurement noise is more than 1° r.m.s. In general, these two situations are constraints to any type of filtering technique.

CONCLUSION

PF (which is useful for nonlinear and non-Gaussian applications) combined with MGBKf is proposed to estimate target motion parameters in passive target tracking. The performance of the PFMGBKf is greatly superior to the standard EKF. In this paper, tracking of torpedo using towed array measurements is explored. Ownship safety maneuver is used for observability of the process. CPA method uses the estimated torpedo motion parameters to find out ownship evasive maneuver. Extensive simulation is carried out, and the results are found to be consistent. For the purpose of presentation, results of three typical scenarios are presented.

REFERENCES