

## RELIABLE ESTIMATION TECHNIQUE FOR UNDERSEA TARGET LOCALIZATION USING ACOUSTIC PROPAGATION FOR MARITIME SIGNAL PROCESSING APPLICATIONS

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### ABSTRACT

**Objective:** The feasibility of a novel transformation, known as unscented transformation, which is designed to propagate information in the form of mean vector and covariance matrix through a nonlinear process, is explored for underwater applications.

**Methods:** The unscented transformation coupled with certain parts of the classic Kalman filter, provides a more accurate method than the EKF for nonlinear state estimation. Using bearings only measurements, Unscented Kalman filter algorithm estimates target motion parameters and detects target maneuver, using zero mean Chi-square distributed random sequence residuals, in sliding window format. During the period of target maneuvering, the covariance of the process noise is sufficiently increased in such away that, the disturbances in the solution is less.

**Results:** When target maneuver is completed, the covariance of process noise is lowered. In seawater, targets move at different speeds and will be at different ranges. It is observed that this algorithm is able to track all types of targets with encouraging convergence time.

**Conclusion:** The performance of this algorithm is evaluated in Monte Carlo simulation and results are shown for various typical geometries.

**Keywords:** Estimation, Target tracking, Simulation, Sonar, Kalman filter, Bearing, Range, Unscented Kalman filter.

### INTRODUCTION

In the ocean environment, two-dimensional bearings only target motion analysis (TMA) is generally used. An observer monitors noisy sonar bearings from a radiating target in passive listening mode and processes these measurements to find out target motion parameters, viz., range, course, bearing, and speed of the target. As range measurement is not available and the bearing measurement is not linearly related to the target states, the whole process becomes nonlinear [1-3]. Added to this, since bearing measurements are extracted from single passive sonar, the process remains unobservable until observer executes a proper maneuver. For presenting the concepts in clear, it is assumed here that the target is moving at constant velocity. The traditional Kalman filter is optimal when the model is linear. Unfortunately, many of the state estimation problems such as the above-mentioned tracking of the target using bearings only information, is nonlinear, thereby limiting the practical usefulness of the Kalman filter and Extended Kalman filter [4,5]. Hence, the feasibility of a novel transformation, known as unscented transformation, which is designed to propagate information in the form of mean vector and covariance matrix through a nonlinear process, is explored for underwater applications. The unscented transformation coupled with certain parts of the classic Kalman filter provides a more accurate method than the EKF for nonlinear state estimation. It is more accurate, easier to implement and uses the same order of calculations. So far in the literature, the target is assumed to be moving with a constant velocity. When the target carries out a maneuver, the solution diverges. Then, the usual practice for underwater applications is to restart the whole process, on the detection of target maneuver by the innovation algorithm. In this paper, Unscented Kalman filter (UKF) [4,5] algorithm is tried out to track the maneuvering target, using bearings only measurements, which are available from passive sonar.

The target observer geometry is given in Fig. 1. The detection of target maneuver is carried out as follows. In this process, it is assumed that the estimator UKF is of high quality in the sense that solution is possible for all tactical scenarios including all quadrants (several geometries

are tested using UKF, and the solution is invariably obtained). It is also assumed that the solution diverges only when the target maneuvers.

When the target is not maneuvering, it is observed from many geometries that the bearing residuals of UKF are almost zero and their small scatter around the zero bearing line is the random noise. It is also noted that the bearing residuals are not close to zero when the target is maneuvering. It is very difficult to confirm whether the target has maneuvered or not just by visual inspection of the bearing residual plot, due to the corruption of the bearing measurements with random noise. Hence, zero mean Chi-square distributed random sequence residuals, in sliding window format, are used for the detection of target maneuver. Target maneuver is declared when the normalized squared innovations exceed the threshold. Sufficient plant noise is inputted to the covariance matrix during the period of target maneuver so that the disturbances in the solution is less. When the maneuver is completed, i.e., the normalized squared innovations is less than the threshold, the process noise level is lowered.

The paper is organized as follows. Section 2 describes mathematical modeling of the measurements, observer, and target motions. It also describes the formulation of UKF for bearings only target motion analysis, deals with target maneuver detection algorithm using normalized squared innovation process and also with the adjustment of covariance matrix during target maneuver. In section 3, implementation aspects such as initialization of state vector and simulation are discussed. The results in Monte Carlo simulation for various scenarios using UKF are presented. Limitations of the algorithm are discussed in section 4. Finally, the paper is concluded in section 5. It is well-known that particle filter (PF) takes more time than that of UKF and UKF itself is able to generate the required accuracies in the estimated solution, PF is not discussed in this paper.

### MATHEMATICAL MODEING

#### State and measurement equations

Let the target state vector be  $X_s(k)$  where,

$$X_s(k) = [\dot{x}(k) \quad \dot{y}(k) \quad R_x(k) \quad R_y(k)]^T \quad (1)$$

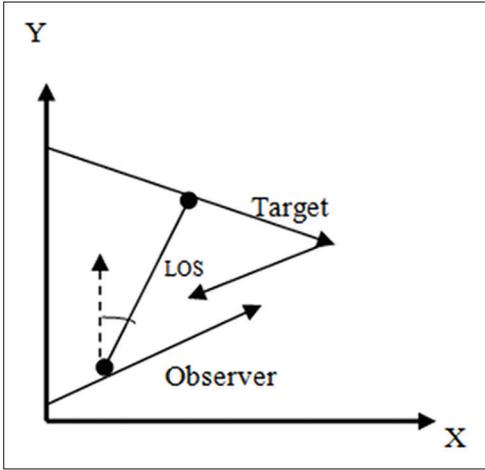


Fig. 1: Target and observer encounter

Where,  $\dot{x}(k)$  and  $\dot{y}(k)$  are target velocity components, and  $R_x(k)$  and  $R_y(k)$  are range components, respectively.

The target state dynamic equation is given by,

$$X_s(k+1) = \phi X_s(k) + b(k+1) + \Gamma \omega(k) \tag{2}$$

Where,  $\phi$  and  $b$  are transition matrix and deterministic vector, respectively.

The transition matrix is given by,

$$\phi = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ t & 0 & 1 & 0 \\ 0 & t & 0 & 1 \end{bmatrix}$$

Where  $t$  is sample time,

$$b(k+1) = \begin{bmatrix} 0 & 0 & -\{x_0(k+1) - x_0(k)\} & -\{y_0(k+1) - y_0(k)\} \end{bmatrix}$$

$$\text{and } \Gamma = \begin{bmatrix} t & 0 \\ 0 & t \\ \frac{t^2}{2} & 0 \\ 0 & \frac{t^2}{2} \end{bmatrix} \tag{3}$$

Where,  $x_0$  and  $y_0$  are observer position components. The plant noise,  $\omega(k)$  is assumed to be zero mean white Gaussian with  $E[\omega(k)\omega(j)] = Q\delta_{kj}$ . True North convention is followed for all angles to reduce mathematical complexity and for easy implementation. The bearing measurement,  $B_m$  is modeled as,

$$B_m(k+1) = \tan^{-1} \left( \frac{R_x(k+1)}{R_y(k+1)} \right) + \zeta(k) \tag{4}$$

Where,  $\zeta(k)$  is error in the measurement and this error is assumed to be zero mean Gaussian with variance  $\sigma^2$ . The measurement and plant noises are assumed to be uncorrelated to each other. Equation (4) is a nonlinear equation.

The plant noise  $\omega(k)$  is assumed to be zero mean white Gaussian. It is given by:

$$\omega(k) = \begin{bmatrix} \omega_{\dot{x}} \\ \omega_{\dot{y}} \end{bmatrix} \tag{5}$$

The plant noise covariance matrix is given by:

$$Q(k) = \begin{bmatrix} ts^2 & 0 & \frac{ts^3}{2} & 0 \\ 0 & ts^2 & 0 & \frac{ts^3}{2} \\ \frac{ts^3}{2} & 0 & \frac{ts^4}{4} & 0 \\ 0 & \frac{ts^3}{2} & 0 & \frac{ts^4}{4} \end{bmatrix} * d(k) \tag{6}$$

Where,  $d(k)$  is given by:

$$d(k) = E[\omega(k)\omega^T(k)]$$

**Unscented Kalman filter (UKF) algorithm**

The state equation is given by:

$$X(k+1) = F(X(k), \phi(k)) + W(k) \tag{7}$$

where  $w_k$  is the plant noise. The unscented Kalman filter (UKF) uses  $(2n+1)$  scalar weights (mean and covariance), which can be calculated as:

$$W_0^{(m)} = \frac{\lambda}{n+\lambda}$$

$$W_0^{(c)} = \frac{\lambda}{n+\lambda} + \beta$$

$$W_i^{(m)} = W_i^{(c)} = \frac{1}{2(n+\lambda)} \quad i = 1, 2, \dots, 2n \tag{8}$$

Where,  $\lambda = (\sigma^2 - 1) \alpha$  is a scaling parameter,  $\alpha$  determines the spread of the sigma points around the mean  $\bar{x}$  and is usually set to a small positive value and  $\beta$  is used to incorporate prior knowledge of the state distribution  $x$  (for Gaussian distribution,  $\beta=2$  is optimal). The standard UKF implementation consists of the following steps:

1. Calculation of the  $(2n+1)$  sigma points starting from the initial conditions  $x(k) = x(0)$  and  $P(k) = P_0$ .

$$X(k) = \begin{bmatrix} x(k) & x(k) + \sqrt{(n+\lambda)P(k)} & x(k) - \sqrt{(n+\lambda)P(k)} \end{bmatrix} \tag{9}$$

2. Transformation of these sigma points through the process model using equation (12).
3. The prediction of the state estimate at time  $k$  with measurement up to time  $k+1$  is given as:

$$x(k+1|k) = \sum_{i=0}^{2n} W_i^{(m)} x(i, k+1|k) \tag{10}$$

As the process noise is additive and independent, the predicted covariance is given as:

$$P(k+1|k) = \sum_{i=0}^{2n} W_i^{(c)} \left[ x(i, k+1|k) - x(k+1|k) \right] \left[ x(i, k+1|k) - x(k+1|k) \right]^T + Q(k) \tag{11}$$

4. Updation of the sigma points with the predicted mean and covariance. The updated sigma points are given as:

$$X(k+1|k) = \begin{bmatrix} x(k+1|k) & x(k+1|k) + \sqrt{(n+\lambda)P(k+1|k)} & x(k+1|k) - \sqrt{(n+\lambda)P(k+1|k)} \end{bmatrix} \tag{12}$$

5. Transformation of each of the predicted points through measurement equation
6. Prediction of measurement (innovation), given as:

$$y(k+1|k) = \sum_{i=0}^{2n} W_i^{(m)} Y(i, k+1|k) \tag{13}$$

7. Since the measurement noise is also additive and independent, the innovation covariance is given as:

$$P_{yy} = \sum_{i=0}^{2n} W_i^{(c)} [Y(i, k+1|k) - y(k+1|k)] \cdot [Y(i, k+1|k) - y(k+1|k)]^T + R(k) \tag{14}$$

8. The cross covariance is given as:

$$P_{xy} = \sum_{i=0}^{2n} W_i^{(c)} [X(i, k+1|k) - x](k+1|k) \cdot [Y(i, k+1|k) - y(k+1|k)]^T \tag{15}$$

9. Kalman gain is calculated as:

$$K(k+1) = P_{xy} \cdot P_{yy}^{-1} \tag{16}$$

10. The estimated state is given as:

$$X(k+1|k+1) = X(k+1|k) + K(k+1)(y(k+1|k+1) - y(k+1|k)) \tag{17}$$

Where,  $y(k)$  is true measurement.

11. Estimated error covariance is given as:

$$P(k+1|k+1) = P(k+1|k) - K(k+1) \cdot P_{yy} \cdot K(k+1)^T \tag{18}$$

**Target maneuver detection**

In under water, the target moves, in general, at constant speed and occasionally maneuvers by changing its course so that it reaches its destination, at a reasonable time, without being caught by the observer. If the target moves at constant velocity, it is easily caught by the observer.

The observer, in passive mode, has only target bearing information. Hence, it makes maneuvers so that it can obtain the target motion parameters. In general, observer's one maneuver is sufficient to make the process observable. As the noise is present, maximum number of observer's maneuvers are required to obtain the acceptable solution. The maneuver command can be modeled as a random process or a deterministic process. The random process is further classified as white noise or autocorrelated noise, according to the statistical characteristics of the process modeling of the maneuver. In long range sonar applications, we can ignore the details of a maneuver and concentrate on detecting its occurrence. Here, the target maneuver is modeled as a random process and accordingly the post-maneuvering estimate is corrected. In the modeling of the dynamics of nonmaneuvering targets, the process noise is assumed to be low. A maneuver manifests itself into a large innovation.

A fudge factor is used to scale up the process noise such that the modified prediction covariance is sufficiently large when the maneuver has taken place. The process noise level is lowered after the completion of the target maneuver. The normalized innovations squared is given by [6].

$$\gamma_\phi(k+1) = \phi^T(k+1) S^{-1}(k+1) \phi(k+1) \tag{19}$$

Where,  $\phi(k+1)$  is innovation or measurement residual and is given by,

$$\phi(k+1) = B_m(k+1) - h(k+1, X(k+1|k)) \tag{20}$$

Let  $s(k)$  be covariance matrix of  $\phi(k)$ .  $s(k)$  is given by,

$$S(k+1) = H(k+1)P(k+1|k)H^T(k+1) + \sigma^2 \tag{21}$$

The values of  $s(k)$  for  $k = 1, 2, \dots$  will be taken as the diagonal elements in the matrix  $S$ . For example, if window size 5,

$$S = [s(1) \ 0 \ 0 \ 0 \ 0; \ 0 \ s(2) \ 0 \ 0 \ 0; \ 0 \ 0 \ s(3) \ 0 \ 0; \ 0 \ 0 \ 0 \ s(4) \ 0; \ 0 \ 0 \ 0 \ 0 \ s(5)];$$

A maneuver is declared only if  $d(\xi)$ , as defined in Equation (14) is statistically significant. That is

$$d(\xi) = \gamma^T S^{-1} \gamma \geq c \tag{22}$$

$$\text{Where, } v = [\varphi(1) \ \varphi(2) \ \dots \ \varphi(k)]^T \tag{23}$$

Where  $c$  is threshold. The statistic  $d$  is a Chi-square distributed with  $n$  degrees of freedom (where  $n$  is equal to dimension of the sliding window). The size of the sliding window, in maneuver detection, is selected on the basis of the results of several geometries. In general, the window size of at least five samples is taken so that the reliability is increased in highly noisy environment prevalent in underwater scenario. The higher the window size, the higher is the value of  $d(\xi)$ . This window size is chosen as six, based on the results of Monte Carlo simulation against a number of geometries. The details of the simulation are as follows.

**IMPLEMENTATION AND SIMULATION**

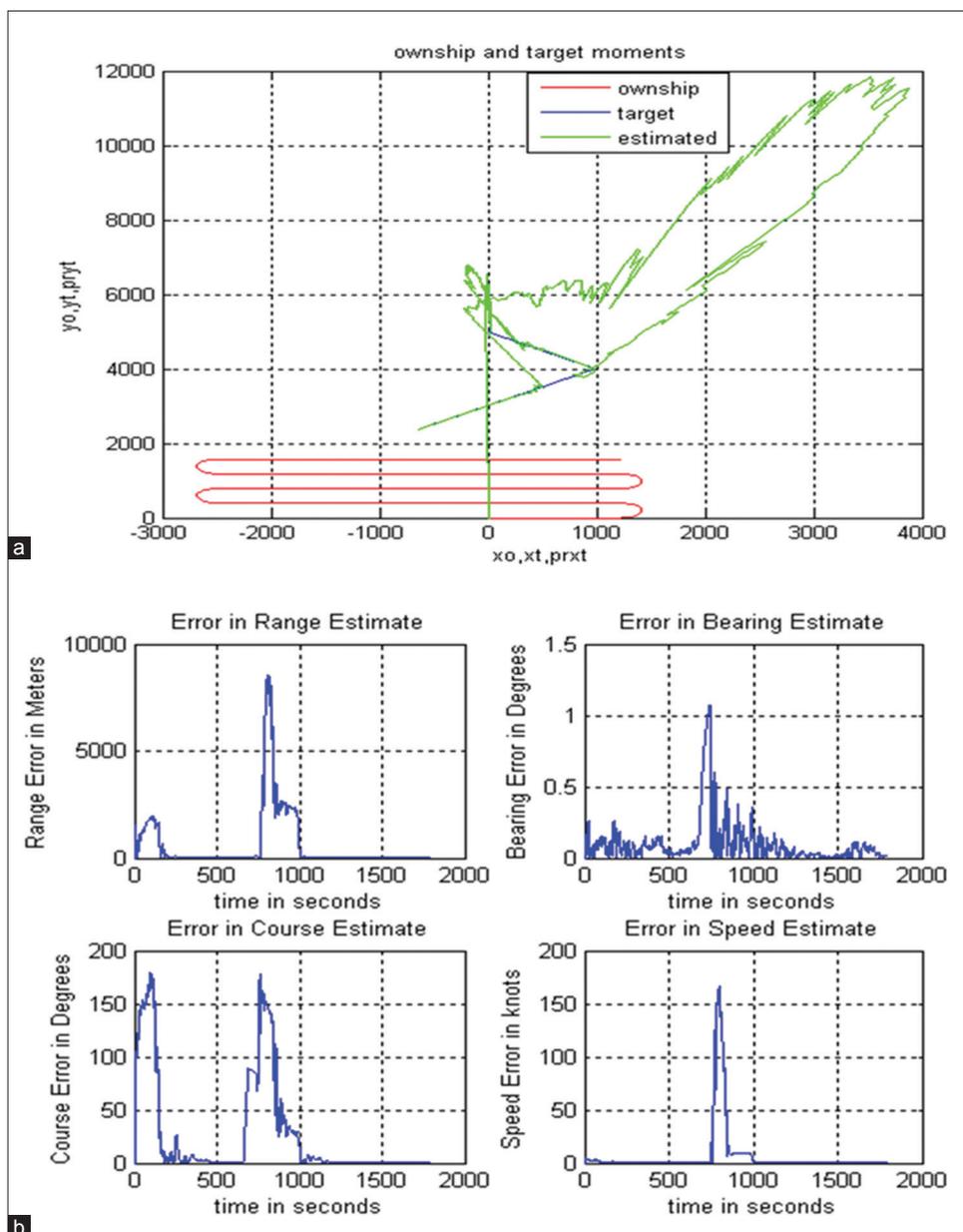
All raw bearings and frequency measurements are corrupted by additive zero mean Gaussian noise. The performance of this algorithm is evaluated against number of geometries. For the purpose of presentation, the results of the scenario considered as shown in Table 1 (Scenario 1 is for ship and the Scenario 2 is for submarine).

The measurement interval is 1 second and the period of simulation is 1800 seconds. Here, all angles are considered with respect to True North 0-360°, clockwise positive.

All 1 second samples are corrupted by additive zero mean Gaussian noise with a r.m.s level of 0.33°. The observer is assumed to be doing 'S' maneuver on the line of sight at a given constant speed and at a turning rate of 3°/second. The observer moves initially at 90° course for 2 minutes, and then, it changes its course to 270°. At 9<sup>th</sup>, 16<sup>th</sup>, and 23<sup>rd</sup> minutes, the observer changes its course from 270 to 90, 90 to 270, and 270 to 90° degrees, respectively. It is also assumed that the bearing measurements are available continuously every second. A number of scenarios are tested by changing the course of the target in steps of 3° in such a way that the angle between the target course and line of sight is always <60°, as only closing targets are of interest to the observer. The results of these scenarios in Monte Carlo simulation are noted and it is found that the observability in the target motion parameters has taken place after the completion of the observer's first maneuver. In general, the error allowed in the estimated target motion parameters in underwater are ten percent in range, 3° in course and 4 m/second in velocity estimates. Around 80% required solution is realized after observer's second maneuver and 90 to 95% required solution is realized after the third. From the results, it is observed that the solution with required accuracy is obtained from 6<sup>th</sup> minute onward.

**Table 1: Scenarios considered**

Parameter		Scenario 1	Scenario 2
Initial range (m)		5000	5000
Initial bearing (°)		0	0
Target speed (m/seconds)	2		2
Target course (°)		135	135
Observer speed m/second	10		3
Observer course (°)		90	
Error in the bearing (one sigma) (°)	0.33		0.33



Scenario 1: (a) Simulated and true target paths. (b) Errors in the target motion parameters

The theoretical value of Chi-square variable with 5° of freedom at 90% confidence level is 9.24. The higher value of  $d(\xi)$  is due to the consideration of EKF using Taylor series expansion up to the first order. When there is a target maneuver it is changing from around 200-500 in 20 seconds initially and afterward in next 4-5 seconds, it is changing more than 2000. In this scenario, the observer moves from the origin in S - maneuver on initial line of sight at 2.0 m/second with a turning rate of 3°/second. After the completion of four maneuvers, it maintains 90° course throughout the simulation period. It is assumed that the target is maneuvering from 135° to 235° with a turning rate of 3°/second at 660 seconds. The target has maneuvered from 135 to 235° at 660 seconds. Target maneuver is declared when the normalized squared innovations exceed the threshold. Sufficient state noise is inputted to the covariance matrix during the period of target maneuver so that the filter comes back to steady state experiencing only acceptable disturbance during target maneuvering period. When the maneuver is completed, i.e., when the normalized squared innovations are less than the threshold, the process noise level is lowered to 1.

For the implementation of the algorithm, the initial estimate of the target state vector is chosen as follows. As range measurements are not

available, it is difficult to guess the velocity components of the target. Hence, these components are each assumed as 5 m/second, which are roughly close to the general speeds of the vehicles in underwater. The sonar range of the day, say 5000 m, is utilized in the calculation of initial position components of the target state vector as follows.

The target state vector is:

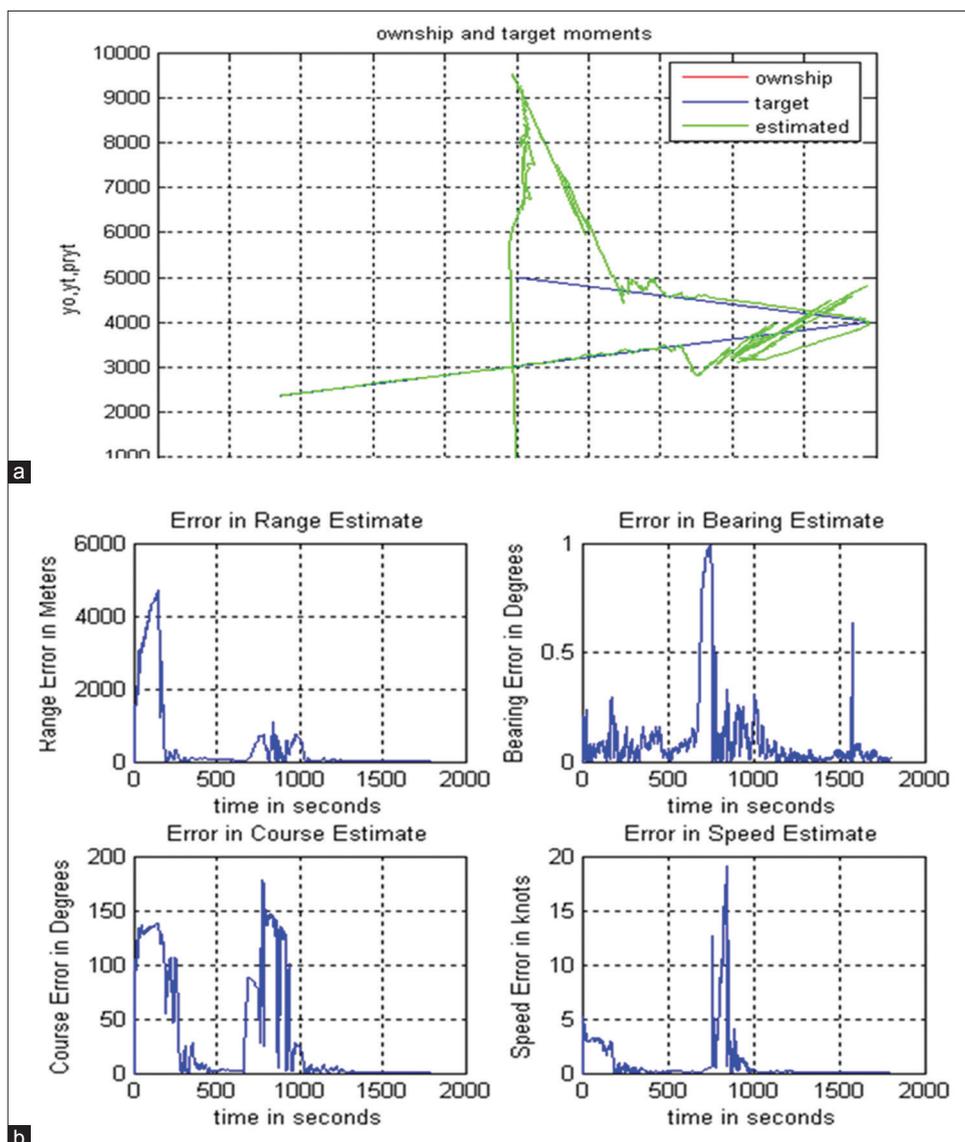
$$X_s(0) = [5 \quad 5 \quad 5000 \sin B_m(0) \quad 5000 \cos B_m(0)]^T$$

Where  $B_m(0)$  is the initial bearing measurement.  $\omega_x(k)$  and  $\omega_y(k)$  are the disturbances in acceleration component along X and Y-axis. The initial covariance matrix  $P(0/0)$  is a diagonal matrix with the elements are given by,

$$P(0/0) = \text{Diagonal} \left( 4 * X_s^{(a)}(i)^2 / 12 \right) \quad \text{Where } i = 1, 2, \dots, 4$$

**LIMITATIONS OF THE ALGORITHM**

Angle on target bow (ATB) is the angle between the target course and line of sight. When ATB is more than 60°, the distance between



Scenario 2: (a) Estimated and true target paths, (b) Errors in the target motion parameters

the target and observer increases as time increases and hence bearing rate decreases. The algorithm cannot provide good results when the target is going away or the measurement noise is more than  $1^\circ$  rms.

In general, the sonar can listen to a target when SNR is sufficiently high. When SNR becomes less, auto tracking of the target fails, the sonar tracks the target in manual mode and the measurements are not available continuously. The bearings available in manual mode are highly inconsistent and are not useful for good tracking of the target. In underwater, it is also possible that sonar measurement sometimes is spurious (the difference between the present and previous measurement being very high) and the same is treated as invalid. In this algorithm, it is assumed that good track continuity is maintained over the simulation period. This means that propagation conditions are satisfactory during this period as well as track continuity is maintained during ownship maneuvers. The algorithm cannot provide good results when the measurement noise is more than  $1^\circ$  rms or when the target is going away w.r.t. the observer.

## SUMMARY AND CONCLUSION

The authors have tried UKF for bearings only underwater maneuver target tracking in Monte Carlo simulation and observed that the results are satisfactory. Hence, UKF is recommended for this application.

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