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Review Article

A NOVEL APPROACH TO STATE SPACE TIME DOMAIN AUTOREGRESSIVE SIGNAL PROCESSING USING OPTIMAL RECURSIVE ESTIMATOR

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ABSTRACT

This work describes the concept of filtering of signals using discrete Kalman filter. The true state of constant, random constant having process noise and autoregressive (p) process when corrupted by measurement noise are estimated using discrete Kalman filter and results are presented using MATLAB.

Keywords: Discrete kalman filter, Noise, True state.

INTRODUCTION

Filtering is desirable in many situations in engineering and embedded systems. For example, radio communication signals are corrupted with noise. A good filtering algorithm can remove the noise from electromagnetic signals while retaining the useful information [1].

If the statistics of the signal are known beforehand, an optimum filter can be designed according to the Wiener -Hopf equations. The drawback of this approach is that in the real world the signals input to the filter are not stationary. Under such circumstances, adaptive filters must be designed, to track the changes of signal and noise [2].

The Newton and steepest descent algorithms are investigated as possible searching methods for adaptive filtering. Although both methods are not directly applicable to practical adaptive filtering, smart reflections inspired them which led to practical algorithms such as least mean squares (LMS) and Newton-based algorithms.

The LMS is a search algorithm in which a simplification of gradient vector computation is made possible by appropriately modifying the objective function. The convergence speed of the LMS is dependent on the eigenvalue spread of the input signal autocorrelation matrix. The normalized LMS algorithm is simpler to use than the LMS algorithm because it allows the step size to be selected without having to know the largest eigenvalue of the autocorrelation matrix of input signal. It simplifies the selection of step size to ensure that the coefficients converge.

The recursive least squares algorithm ensures fast convergence even when the eigenvalue spread of the input signal autocorrelation matrix is large. All these advantages come with the cost of an increased computational complexity and some stability problems, which are not as critical in LMS based algorithms [3].

In 1960, R.E. Kalman published his famous paper describing a recursive solution to the discrete data linear filtering problem. Since that time, due to large part in advances in digital computing; the Kalman filter has been the subject of extensive research and application, particularly in the area of autonomous or assisted navigation [4,5].

The Kalman filter, rooted in the state space formulation of linear dynamical systems, provides a recursive solution to the linear optimal filtering problem. It applies to stationary as well as non-stationary environments. The solution is recursive in that each updated estimate of the state is computed from the previous estimate and the new input data, so only the previous estimate requires storage. In addition to eliminating the need for storing the entire past observed data, the Kalman filter is computationally more efficient than computing the estimate directly from the entire past observed data at each step of the filtering process [4]. Section 2 presents the state space model and discrete Kalman filter algorithm. Section 3 deals with implementation and simulation. Experimental results are also included in Section 3. Finally, the concluding remarks are presented in Section 4.

MATHEMATICAL MODELING

The Kalman filter is a linear, discrete time, finite dimensional time-varying system that evaluates the state estimate that minimizes the mean-square error. It estimates a process by using a form of feedback control: The filter estimates the process state at some time and then obtains feedback in the form of (noisy) measurements. As such, the equations for the Kalman filter fall into two groups: Time update equations and measurement update equations. The time update equations are responsible for projecting forward (in time) the current state and error covariance estimates to obtain the a priori estimates for the next time step. The measurement update equations are responsible for the feedback – i.e., for incorporating a new measurement into the a priori estimate to obtain an improved a posteriori estimate.

The time update equations can also be thought of as predictor equations, while the measurement update equations can be thought of as *corrector* equations. Indeed the final estimation algorithm resembles that of predictor-corrector algorithm for solving numerical problems as shown in Fig. 1.

State space time domain modelling of auto regressive (AR) process

Consider the system to be governed by the linear constant coefficient difference equation:

$$y(n)=\sum_{k=1}^{M}a(k)y(n-k) \qquad (1)$$

for which the direct form II realization is shown in Fig. 2. Here $w(n)$ is the process noise and $\xi(n)$ is the measurement noise.

The state variables of the system are the numerical quantities memorized by the system that comprise the state. In Fig. 2, $v_1(n),\ldots,v_M(n)$ are the internal variables which comprise the state variables for this system [5].

We have,

$$v_i(n+1)=v_{i+1}(n) \tag{2}$$

$$v_M(n+1)=v_{M+1}(n)=x(n)+w(n)+a(1)v_M(n)+$$
$$a(2)v_{M-1}(n)+\ldots+a(M)v_1(n)=x(n)+w(n)+\sum_{i=1}^{M}a(i)v_{M-i+1}(n) \tag{3}$$

Eq. (2) and Eq. (3) are the state equations for the system.

$$\begin{bmatrix} v_1(n+1)\\ v_2(n+1)\\ v_3(n+1)\\ \vdots\\ \vdots\\ v_{M-1}(n+1)\\ v_M(n+1)\end{bmatrix}=\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & . & . & 0\\ 0 & 0 & 1 & 0 & 0 & . & . & 0\\ 0 & 0 & 0 & 1 & 0 & . & . & 0\\ . & . & . & . & . & . & . & .\\ . & . & . & . & . & . & . & .\\ 0 & 0 & 0 & 0 & 0 & . & . & 1\\ a(M) & a(M-1) & a(M-2) & a(M-3) & a(M-4) & . & . & a(1)\end{bmatrix}\begin{bmatrix} v_1(n)\\ v_2(n)\\ v_3(n)\\ \vdots\\ \vdots\\ v_{M-1}(n)\\ v_M(n)\end{bmatrix}+\begin{bmatrix}0 & 0 & 0 & \cdots & 0 & 1\end{bmatrix}^T\left((x(n)+w(n)\right) \tag{4}$$

$$\Rightarrow v(n+1)=Av(n)+c(x(n)+w(n)) \tag{5}$$

Where

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & . & . & 0\\ 0 & 0 & 1 & 0 & 0 & . & . & 0\\ 0 & 0 & 0 & 1 & 0 & . & . & 0\\ . & . & . & . & . & . & . & .\\ . & . & . & . & . & . & . & .\\ 0 & 0 & 0 & 0 & 0 & . & . & 1\\ a(M) & a(M-1) & a(M-2) & a(M-3) & a(M-4) & . & . & a(1)\end{bmatrix} \tag{6}$$

and $c=[0\ 0\ 0.\ 0\ 1]^T$ (7)

The output can be computed from the state variables at time n using

$$y(n)=v_{M+1}(n)+\xi(n)=x(n)+w(n)+\begin{bmatrix}a(M) & a(M-1) & \cdots\cdots & a(1)\end{bmatrix}\begin{bmatrix}v_1(n)\\ v_2(n)\\ \vdots\\ v_M(n)\end{bmatrix}+\xi(n)=b^Tv(n)+d(x(n)+w(n))+\xi(n) \tag{8}$$

Where $b=[a(M)\ a(M-1)\ a(1)]^T)$ (9)

and d=1 (10)

The discrete Kalman filter algorithm

State Equation

$v(n+1)=A(n)v(n)+C(n)(x(n)+w(n))$

Observation Equation

$y(n)=B(n)v(n)+d(x(n)+w(n))+\xi(n)$

The system noise $w(n)$ and the measurement noise $\xi(n)$ are assumed to be white Gaussian noise with known variances $Qw^{(n)}$ and $Q\xi^{(n)}$ respectively.

Initialization: $\hat{v}(0|0)=E\{v(0)\}$

$P(0|0)=E\{v(0)v^H(0)\}$

Computation: For $n=1,2$ compute

1. Assuming the estimate of state vector $\hat{v}(n/n)$, the error covariance $P(n/n)$ are obtained from n^{th} iteration. The predicted state vector and prediction error covariance matrix at the $(n+1)^{th}$ iteration are then computed from

$$\hat{v}(n+1|n)=A(n)\hat{v}(n|n)$$

$$P(n+1|n)=A(n)P(n|n)A^H(n)+Qw(n+1)$$

2. The Kalman filter gain $K(n+1)$ is then computed from

$K(n+1)=P(n+1|n)B^H(n+1[B(n+1)P(n+1)B^H(n+1+Q_\xi(n+1)]^{-1.}$

3. The estimate of the state vector and the corresponding error covariance matrix are updated after obtaining a new measurement data $y(n+1)$ at time n using.

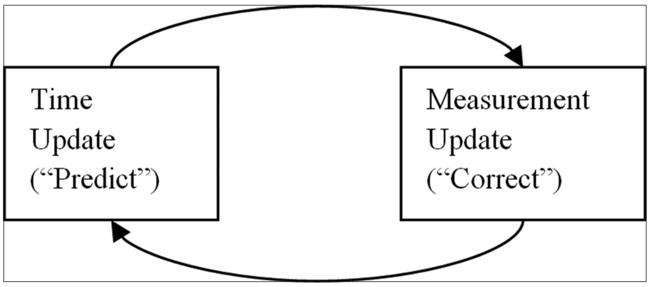

Fig. 1: The ongoing discrete Kalman filter cycle

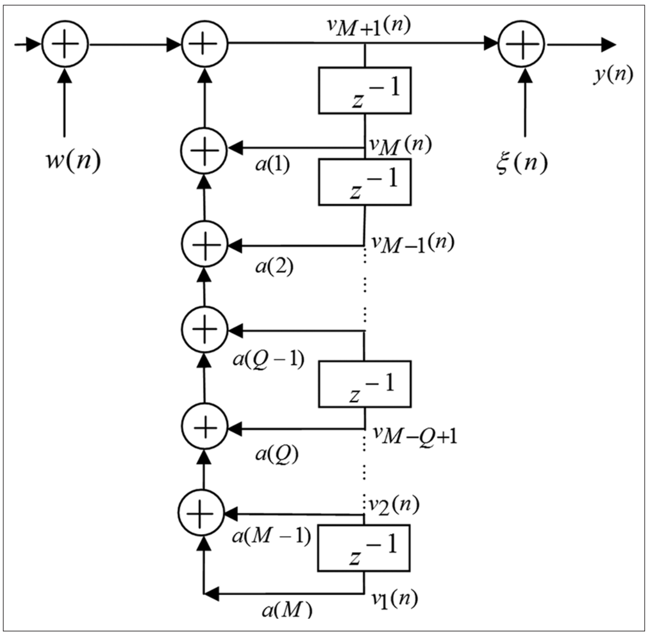

Fig. 2: Direct form II realization of the discrete time system with input-output description

$$\hat{v}(n+1|n+1)=\hat{v}(n+1|n)+K(n+1)\left[Y(n+1)-B(n+1)\hat{v}(n+1|n)\right]$$

$$P(n+1|n+1)=\left[1-K(n+1)B(n+1)\right]P(n+1|n)$$

As the time update projects the current state estimate ahead in time, the measurement update adjusts the projected estimate by an actual measurement at that particular time. The first task during the measurement update is to compute Kalman gain. Next, is to measure the process to obtain the measurement and then generate a posteriori state estimate by incorporating the measurement. Finally, the last step is to obtain the posteriori error covariance estimate. Thus, after each time and measurement update pair, this loop process is repeated to project or predict the new time step priori estimates using the previous time step posteriori estimates. The Kalman filter recursively conditions the current estimate on all of the past measurements. The complete picture of the operation of Kalman filter is shown in Fig. 3.

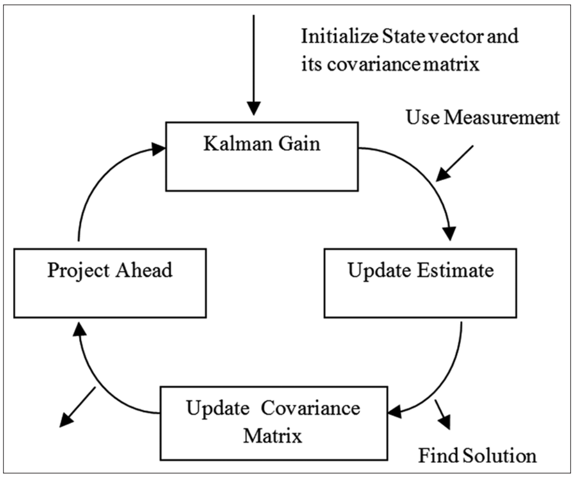

Fig. 3: A complete picture of the operation of the Kalman filter

IMPLEMENTATION AND SIMULATION

Estimating a constant using discrete Kalman filter

1. Example: Let us attempt to estimate a scalar constant x=2, a voltage for example. Let's assume that we have the ability to take measurements of the constant, but that the measurements are corrupted by a √0.0 volt RMS white Gaussian measurement noise.
2. Simulation: The process is governed by the linear difference equation (5) with a measurement given by Eq. (8). The state does not change from step to step so A=1. Though the process noise w=0, a very small process variance of the order of Q_w=0.01 is assumed. Here, the state is nothing but measurement so **C**=1. The variance of measurement noise is considered as Q_ξ=0.1. Let the initial estimate of **v** and error covariance P be 1.50 distinct measurements *y(n)* that had an error normally distributed around zero with a standard deviation of √0.1 is then simulated. Fig. 4 depicts the results of this simulation.

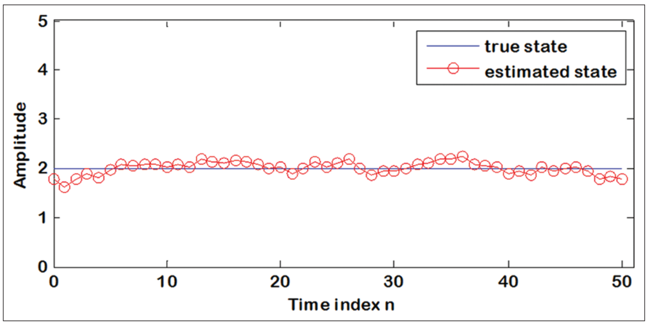

Fig. 4: Estimating a constant using discrete Kalman filter

Estimating a random constant having process noise using discrete Kalman filter

1. Example: Let us attempt to estimate a scalar random constant *x=2* corrupted by √0.1 volt RMS white Gaussian process noise, a voltage for example. The measurements are corrupted by a √0.01 volt RMS white Gaussian measurement noise.
2. Simulation*:* The process is governed by the linear difference equation (5) with a measurement given by Eq. (8). Here, the process noise and measurement noise are considered as white Gaussian noises with variances 0.1 and 0.01 respectively. The state matrix A and the measurement matrix **C** are both taken as 1. Let the initial estimate of **v** and error covariance P be 1.50 distinct measurements *y(n)* that had an error normally distributed around zero with a standard deviation of √0.01 is then simulated. Fig. 5 depicts the results of this simulation.

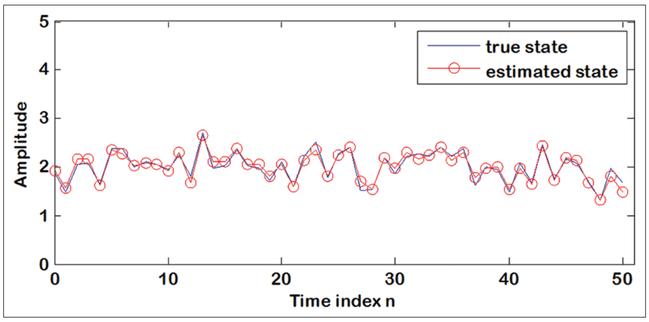

Fig. 5: Estimating a random constant having process noise using discrete Kalman filter

Estimating an AR (p) process using discrete Kalman filter

1. Example*:* Let x(n) be the AR (p) process that is generated by the following difference equation

$$x(n)=\sum_{k=1}^{p} a(k)x(n-k)+w(n) \tag{11}$$

Where *w(n)* is the white Gaussian noise with a variance 0.36, and let

$$y(n)=x(n)+\xi(n) \tag{12}$$

be noisy measurements of *x(n)* and ξ(n) is white Gaussian noise with variance 0.01 that is uncorrelated with w(n).

Let p=4 so the AR (4) process is generated according to the difference equation

$$x(n)=0.1x(n-1)+0.2x(n-2)+0.3x(n-3)+0.4x(n-4)+w(n) \tag{13}$$

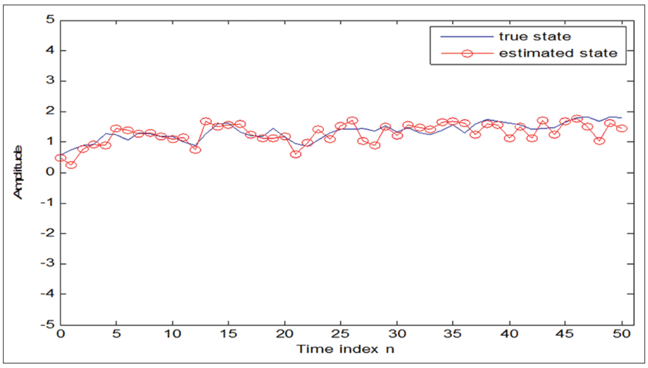

Fig. 6: Estimating an auto regressive (4) process using discrete Kalman filter

The state matrix A is a matrix of order 1×p and the measurement matrix **C** is an identity matrix of order p.

Let the initial estimate of **v** be a zero vector matrix of order 1×p and error covariance P be identity matrix of order p.

2. Simulation*:* The state matrix A is a matrix of order 1×4 and the measurement matrix **C** is an identity matrix of order 4. Let the initial estimate of **v** be a zero vector matrix of order 1×4 and error covariance **P** be identity matrix of order 4. 50 distinct measurements *y(n)* that had an error normally distributed around zero with a standard deviation of √0.01 is simulated. Fig. 6 depicts the results of this simulation.

In all the cases, the true value is given by the solid line and the filter estimate by the remaining curve. Under conditions where the covariance of process noise Q_w and measurement noise Q_ξ are constant, both the estimation error covariance P and Kalman gain K will stabilize quickly and then remain constant. If this is the case, these parameters can be pre-computed by running the filter off-line. It is frequently the case however that the measurement error (in particular) does not remain constant.

CONCLUSION

It is tried to estimate the true state by implementing discrete Kalman filter for different cases and observed that the results are satisfactory.

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