

## RELIABLE ESTIMATION TECHNIQUE FOR UNDERWATER VEHICLE DETECTION AND LOCALISATION USING ACOUSTIC PROPAGATION FOR SONAR APPLICATIONS

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### ABSTRACT

Inclusion of range, course, and speed parameterization is proposed for modified gain bearings only extended Kalman filter to track a torpedo using bearings-only measurements. Parameterization is included to obtain fast convergence in estimated torpedo motion parameters. The ownship is assumed to be under torpedo attack, and the bearing measurements are available from hull-mounted array of the ownship. Observer uses estimated torpedo motion parameters to calculate optimum evasive maneuver. Monte-Carlo simulation is carried out, and the results are presented for typical scenarios with and without parameterization. It is noted that parameterization reduces the time of convergence and the results are satisfactory.

**Keywords:** Kalman filter, Torpedo, Sonar, Estimation, Tracking, Monte-Carlo simulation.

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### INTRODUCTION

Surveillance is the most important facet of maritime warfare and is undertaken by active as well as passive sensors. Active methods of surveillance require acoustic transmissions to be made by the surveillance platform and hence susceptible to interception by others. Hence, in certain tactical situations, it becomes necessary to maintain silence on active mode. In the ocean environment, two-dimensional bearings-only target motion analysis is generally used. An ownship monitors noisy sonar bearings from a radiating target and finds out target motion parameters (TMP) - viz., range, course, bearing, and speed of the target. The basic assumptions are that the target moves at constant velocity most of the time. The ownship motion is unrestricted. The target and ownship are assumed to be in the same horizontal plane. The problem is inherently nonlinear as the measurement is non-linear. The determination of the trajectory of a target solely from bearing measurements is called bearings-only tracking (BOT). The BOT area has been widely investigated [1-4], and numerous solutions for this problem have been proposed.

For presenting the concepts in clear, it is assumed that the target is moving at constant velocity. Classical least squares method and Kalman filter cannot be directly applied. One useful approach is the pseudo-linear estimator (PLE) formulation proposed [1] which lumps the nonlinearities into the noise term, resulting in a linear measurement equation. Here, the measurement matrix contains elements that are functions of noisy bearings and, overall, are correlated with the noise terms of the measurement equation. As a result, the PLE exhibits a bias in the estimated TMP [5]. As it offers a no diverging solution, many times PLE is used as a backup solution along with the modern filtering techniques (which will be discussed shortly). The classical PLE which is in the form of batch processing is converted into sequential processing [6] in such a way that it does not require initialization of target state vector and this feature is very much useful for maritime underwater target tracking. Maximum likelihood estimator (MLE) is found to be a suitable algorithm for passive target tracking applications, by virtue of its characteristics [1]. This is gradient search based algorithm using batch processing of all the available measurements. MLE is asymptotically efficient, consistent, unbiased and its covariance matrix approaches the Cramer-Rao bound for large samples.

Another approach, utilization of extended Kalman filter (EKF) in modified polar (MP) coordinates [7] frame is found to be useful for

this nonlinear application. In this algorithm, the observable and unobservable components of the estimated state vector are automatically decoupled. Such decoupling is shown to prevent covariance matrix ill-conditioning, which is the primary cause of instability. The MP state estimates are asymptotically unbiased. A hybrid coordinate system approach developed by Walter Grossman [8] is also another successful contribution to bearings-only passive target tracking.

Another successful contribution to this field is by Song and Speyer [9]. The divergence in EKF [3,4] is eliminated by modifying the ownship gains. This algorithm is named as modified gain bearings-only extended Kalman filter (MGBEKF). The essential idea behind MGBEKF is that the nonlinearities be "modifiable." This algorithm has some similarities with the pseudo measurement function but it is not the same. In pseudo measurement filter, the gain is a function of past and present measurements. It is to be noted that MGBEKF is based on EKF algorithm, and the gain of the MGBEKF is a function of only past measurements. By eliminating the direct correlation of the gain and measurement noise process in the estimates of MGBEKF, the bias in the estimation is eliminated. A simplified version of the modified gain function is made available by Galkowski and Islam [10]. This version is useful for air applications, where elevation and bearing measurements are available. This algorithm is further modified for underwater target tracking applications [11,12], where bearings-only measurements are available.

The traditional Kalman filter is optimal when the model is linear. Unfortunately, many of the state estimation problems like tracking of the target using bearings only information are nonlinear, thereby limiting the practical usefulness of the Kalman filter and EKF. Hence, the feasibility of a novel transformation, known as unscented transformation, which is designed to propagate information in the form of mean vector and covariance matrix through a non-linear process, is explored for underwater applications. When the unscented transformation is coupled with certain parts of the classic Kalman filter, it is called as unscented Kalman filter (UKF) [13-17]. UKF can be treated as an alternative to MGBEKF.

Particle filters (PF) [18-21] are the new generation of advanced filters, which are useful for nonlinear and non-Gaussian applications. PF or sequential Monte Carlo methods use a set of weighted state samples, called particles, to approximate the posterior probability distribution

in a Bayesian setup. At any point of time, the set of particles can be used to approximate the PDF of the state. As the number of particles increase to infinity, the approximation approaches the true PDF. They provide nearly optimal state estimates in the case of nonlinear and non-Gaussian systems. The PF is a completely nonlinear state estimator. Of course, there is no free lunch. The price that must be paid for the high performance of the PF is an increased level of computational effort.

In this paper, the target is considered as torpedo. The task is to estimate the torpedo motion parameters, while ownship is in attack by a torpedo. After getting the bearings of the torpedo for few seconds (say for the duration of 15 seconds), ownship tries to escape by doing a certain maneuver. This maneuver is based on  $70^\circ$  relative bearing method, which is being used by Navy (Appendix A). Here, this first maneuver is called as ownship safety maneuver. The idea is to escape from the field as early and quick as possible. In general, the ownship tries to increase the speed after turning to the required course. This is required for the ownship to escape from the target as early as possible.

The ownship's subsequent escape maneuvers can be carried out in systematic way, if torpedo's range, bearing, course and speed are known. As these are not available, these are estimated using an algorithm. Here as bearings are only available, ownship safety maneuver will be used for observability of the process. During the safety maneuver, ownship tries to escape in such a way that range between ownship and target becomes maximum with an increase in time. However, for getting solution, it is another way round. Range should decrease to get more bearing rate with increase in time. With this constraint, ownship tries to estimate the torpedo motion parameters to calculate the proper evasive maneuvers at various time instants and escape from torpedo attack.

The authors are motivated by the work presented by "Branko Ristic, Sanjeev Arulampalam, and Neil Gordon" in "Beyond the Kalman Filter- PF for tracking applications" [17]. These scientists divided the range interval of interest into a number of sub-intervals, and each subinterval is dealt with an independent Kalman Filter. They suggested that this method can be extended to course and speed parameterization if prior knowledge of target course and speed, respectively, are vague. Parameterization in initialization reduces the dependence of convergence of the solution on initialization. In underwater scenario, prior knowledge of torpedo range, course, and speed is vague. Under torpedo attack, the aim is to obtain torpedo motion parameters accurately as early as possible to calculate optimum ownship evasive manoeuvre. In this situation, time to obtain convergence has an important role, and this is achieved using parameterization. The inclusion of range, course, and speed parameterization is proposed for MGBEKF to track a torpedo using bearings-only measurements and this algorithm is named as parameterized modified gain bearings-only extended Kalman filter (PMGBEKF). The ownship is assumed to be under torpedo attack, and the bearing measurements are available from hull-mounted array (HMA) of the ownship. Observer uses estimated torpedo motion parameters to calculate an optimum evasive maneuver.

Monte-Carlo simulation is carried out, and the results are presented for typical scenarios. Results are presented with and without parameterization. It is observed that parameterization reduces the time of convergence.

Section 2 describes mathematical modeling of measurements, formulation of algorithm, and initialization of state vector and its covariance. Section 3 is about simulation and results obtained. Limitations of the algorithm are presented in Section 4 and finally the paper is concluded in Section 5.

## MATHEMATICAL MODELLING

### MGBEKF

The alternative derivation of the modified gain function [9] of Song and Speyer's EKF is slightly modified as follows. Let the target state vector be  $X_s(k)$  where

$$X_s(k) = \begin{pmatrix} \dot{x}(k) \\ \dot{y}(k) \\ R_x(k) \\ R_y(k) \end{pmatrix}^T \quad (1)$$

Where,  $\dot{x}(k)$  and  $\dot{y}(k)$  are target velocity components and,  $R_x(k)$  and  $R_y(k)$  are range components respectively. The target state dynamic equation is given by

$$X_s(k+1) = \phi X_s(k) + b(k+1) + \Gamma \omega(k) \quad (2)$$

Where,  $\phi$  and  $b$  are transition matrix and deterministic vector respectively. The transition matrix is given by

$$\begin{matrix} \dot{\epsilon}_1 & 0 & 0 & 0 \\ \dot{\epsilon}_0 & 1 & 0 & 0 \\ \dot{\epsilon}_{t-1} & 0 & 1 & 0 \\ \dot{\epsilon}_0 & t & 0 & 1 \end{matrix}$$

Where  $t$  is sample time

$$b(k+1) = \begin{pmatrix} \dot{x}_0(k+1) - x_0(k) \\ \dot{y}_0(k+1) - y_0(k) \end{pmatrix}^T$$

$$\text{and } G = \begin{matrix} \dot{\epsilon}_t & 0 & 0 \\ \dot{\epsilon}_0 & t & 0 \\ \dot{\epsilon}_{t-2} & 0 & 0 \\ \dot{\epsilon}_2 & 0 & 0 \\ \dot{\epsilon}_0 & t^2 & 0 \\ \dot{\epsilon}_0 & 0 & \frac{t^2}{2} \end{matrix} \quad (3)$$

Where,  $x_0$  and  $y_0$  are ownship position components. The plant noise  $\omega(k)$  is assumed to be zero mean white Gaussian with  $E[\omega(k) \omega(j)^T] = Qd_{kj}$ . True North convention is followed for all angles to reduce mathematical complexity and for easy implementation. The bearing measurement,  $B_m$  is modeled as

$$B_m(k+1) = \tan^{-1} \frac{\alpha R_x(k+1)}{\alpha R_y(k+1)} + \zeta(k) \quad (4)$$

Where,  $\zeta(k)$  is error in the measurement and this error is assumed to be zero mean Gaussian with variance  $\sigma^2$ . The measurement and plant noises are assumed to be uncorrelated to each other. Equation (4) is a nonlinear equation and is linearized by using the first term of the Taylor series for  $R_x$  and  $R_y$ . The measurement matrix is obtained as:

$$H(k+1) = \begin{pmatrix} 0 & \hat{R}_y(k+1|k)/R^2(k+1|k) \\ 0 & -\hat{R}_x(k+1|k)/R^2(k+1|k) \end{pmatrix} \quad (5)$$

Since the true values are not known, the estimated values of  $R_x$  and  $R_y$  are used in the above equation. The covariance prediction is:

$$P(k+1|k) = \phi(k+1|k)P(k|k)\phi(k+1|k)^T + Q(k+1)\Gamma^T \quad (6)$$

The Kalman gain is

$$G(k+1) = P(k+1|k)H^T(k+1)[\sigma^2 + H(k+1)P(k+1|k)H^T(k+1)]^{-1} \quad (7)$$

The state and its covariance corrections are given by

$$X(k+1|k+1) = X(k+1|k) + G(k+1)[B_m(k+1) - h(k+1, X(k+1|k))] \quad (8)$$

Where,  $h(k+1, X(k+1|k))$  is the bearing using predicted estimate at time index  $k+1$

$$P(k+1|k+1) = [I - G(k+1)g(B_m(k+1), X(k+1|k))]P(k+1|k)[I - G(k+1)g(B_m(k+1), X(k+1|k))]^T + \sigma^2 G(k+1)G^T(k+1) \quad (9)$$

Where,  $g(.)$  is modified gain function as defined in [8]. The value of  $g$  is:

$$\begin{matrix} \dot{\epsilon}_0 & 0 & \cos B_m / (\hat{R}_x \sin B_m + \hat{R}_y \cos B_m) \\ 0 & -\sin B_m / (\hat{R}_x \sin B_m + \hat{R}_y \cos B_m) \end{matrix} \quad (10)$$

Since the true bearing is not available in practice, it is replaced by the measured bearing to compute the function g(.)

### PMGBEKF

The ownship is assumed to be under torpedo attack, and the bearing measurements are available from HMA of the ownship. In underwater scenario, prior knowledge of torpedo range, course and speed is vague. Under torpedo attack, the aim is to obtain torpedo motion parameters accurately as early as possible to calculate optimum ownship evasive manoeuvre. Time to obtain convergence greatly depends on the accuracy of the initialization of the target state vector. Parameterization in initialization reduces the dependence of convergence of the solution on initialization. Inclusion of range, course and speed parameterization is proposed for MGBEKF to track a torpedo using bearings-only measurements. Observer uses estimated torpedo motion parameters to calculate optimum evasive maneuver.

The basic idea is to use a number of independent MGBEKF trackers in parallel, each with a different initial estimate. To do so, the range, course and speed interval of interest is divided into a no of subintervals, and each subinterval is dealt with an independent MGBEKF. Let us suppose the range, course, and speed intervals of interest is ( $\text{range}_{\min}$ ,  $\text{range}_{\max}$ ), ( $\text{course}_{\min}$ ,  $\text{course}_{\max}$ ), and ( $\text{speed}_{\min}$ ,  $\text{speed}_{\max}$ ), respectively, with range, course, speed subintervals. The subintervals are 100 m,  $1^\circ$ , 1 m/seconds in range, course, and speed, respectively. Let

$$\begin{aligned} a &= (\text{range}_{\max} - \text{range}_{\min})/100 \\ b &= (\text{course}_{\max} - \text{course}_{\min})/1 \\ b &= (\text{speed}_{\max} - \text{speed}_{\min})/1 \end{aligned} \quad (11)$$

So number of filters, N is given by ( $a * b * c$ ). The initial weights of each MGBEKF are set to  $1/N$ . Subsequently, the weight of filter  $i$  at time  $k$  is given by

$$w^i(k) = \frac{p(B(k)|i)w^i(k-1)}{\sum_{i=1}^N p(B(k)|i)w^i(k-1)} \quad (12)$$

Where,  $p(B(k)|i)$  is the likelihood of measurement  $B(k)$ . Assuming Gaussian statistics, the likelihood  $p(B(k)|i)$  can be computed as

$$p(B(k)|i) = \frac{1}{\sqrt{2ps_{inv}^i}} \exp\left(-\frac{\hat{B}^i(k|k-1) - B^i(k|k-1)}{s_{inv}^i}\right) \quad (13)$$

Where,  $\hat{B}^i(k|k-1)$  is the predicted angle at  $k$  for filter  $i$ , and  $s_{inv}^i$  is the innovation variance for filter  $i$  given by

$$s_{inv}^i = \hat{H}^i(k)P^i(k|k-1)\hat{H}^{i^T}(k) + s^2 \quad (14)$$

The combined estimate of PMGBEKF is computed using the Gaussian mixture formulas [17].

### Initialization of state and its covariance matrix

In general, the torpedo will be fired at intercept course. With respect to the ownship, the torpedo course will be within (initial bearing  $+/-60^\circ$ ). Hence, the search space in the bearing is reduced to initial bearing  $\pm 60^\circ$ . It is assumed that HMA of the ownship can track the target in auto-tracking mode from 5000 m (maximum range) onward and the ownship speed is in the range of 20-5 m/seconds. The maximum filters required are around 9000, which is found through evaluation of several tactical scenarios. Here, subinterval size is chosen in such away that number of filters is not too high and at the same the accuracy of the solution is obtained within the required time. The initial target state vectors are in the following form:

$$X_s(0/0) = [r * \sin(tcr) \ r * \cos(tcr) \ v * \sin(B_m(0)) \ v * \cos(B_m(0))]^T$$

Where r, initial range in meters=[5000, 4900, 4800, ..., 1000].

$$\text{tcr, torpedo course in degrees} = [B_m(0) + 60 \ B_m(0) - 1] + 60 \ B_m(0) - 2 + 60 \dots \ B_m(0) - 60 \quad (15)$$

$$v, \text{torpedo speed in m/seconds} = [20, 19, 18, \dots, 5] \quad (15)$$

Hence, there will be  $a * 120 * c$ , MGBEKF with different initialization of state vectors. It is assumed that the state vector  $X_s(0/0)$  is uniformly distributed. Then, the element of initial covariance matrix is a diagonal matrix and can be written :

$$P(0/0) = \text{Diag} \begin{bmatrix} \frac{4 * \dot{x}^2(0/0)}{12} & \frac{4 * \dot{y}^2(0/0)}{12} & \frac{4 * \dot{x}_x^2(0/0)}{12} & \frac{4 * \dot{r}_y^2(0/0)}{12} \end{bmatrix} \quad (16)$$

### SIMULATION

In simulation, it is assumed that HMA auto detection range interval is 5000-500 m. Estimation of torpedo motion parameters is stopped when the range is  $\leq 500$  m. Observer maximum speed is 18 knots and turning rate is  $1^\circ/\text{seconds}$ . All 1 second samples are corrupted by additive zero mean Gaussian noise with r.m.s level of  $0.33^\circ$ . The ownship is assumed to be doing safety maneuver with turning rate of  $1^\circ/\text{seconds}$ . The measurement interval is 1 second and the period of simulation is 500 s. All angles are considered with respect to True North 0-360°, clockwise positive. For the purpose of presentation, two scenarios as shown in Table 1 are considered for evaluation of the algorithm. In the figures, the error in estimated range, course and speed are represented as R-error, C-error and S-error, respectively. The results of PMGBEKF after 100 Monte-Carlo runs are shown in Fig. 1a-c for Scenario 1 and Fig. 2a-c for Scenario 2. For the purpose of comparison, the results with one MGBEKF are shown in Fig. 3a-c for scenario 1. In general in underwater scenario, some measurements are outliers. Hence, in simulation 5% of measurements (randomly chosen) are made corrupted with 20 times of than that of expected error, and the corresponding results are shown in Fig. 4a-c for Scenario 1 and Fig. 5a-c for Scenario 2.

It is assumed that the estimated torpedo motion parameters are said to be converged when

- a. Error in the range estimate  $\leq 20\%$  of the actual range
- b. Error in the course estimate  $\leq 5^\circ$ .
- c. Error in the speed estimate  $\leq 4$  knots.

As per the required accuracies, the entire solutions of the estimated parameters range (R), course (C) and speed (S) are shown in Table 2.

From the results, it is very clear that convergence time is reduced (which is very much required in torpedo tracking) using PMGBEKF when compared to with that of one MGBEKF. It is also noted from the results that the algorithm is able to manage satisfactorily with outliers.

### LIMITATIONS OF THE ALGORITHM

In general, the sonar can listen to a target when SNR is sufficiently high. When SNR becomes less, auto tracking of the target fails, the sonar tracks the target in manual mode and the measurements

**Table 1: Details of scenarios chosen**

S. No.	Parameter	Scenario 1	Scenario 2
1	Initial range (meters)	3500	4500
2	Initial bearing (degrees)	75	280
3	Torpedo speed (meters/seconds)	15.45	15.45
4	Target course (degrees)	170	45
5	Observer speed (meters/seconds)	6.18	7.73
6	Observer course (degrees)	0	40
7	r.m.s. error in the bearing (degrees)	0.33	0.33

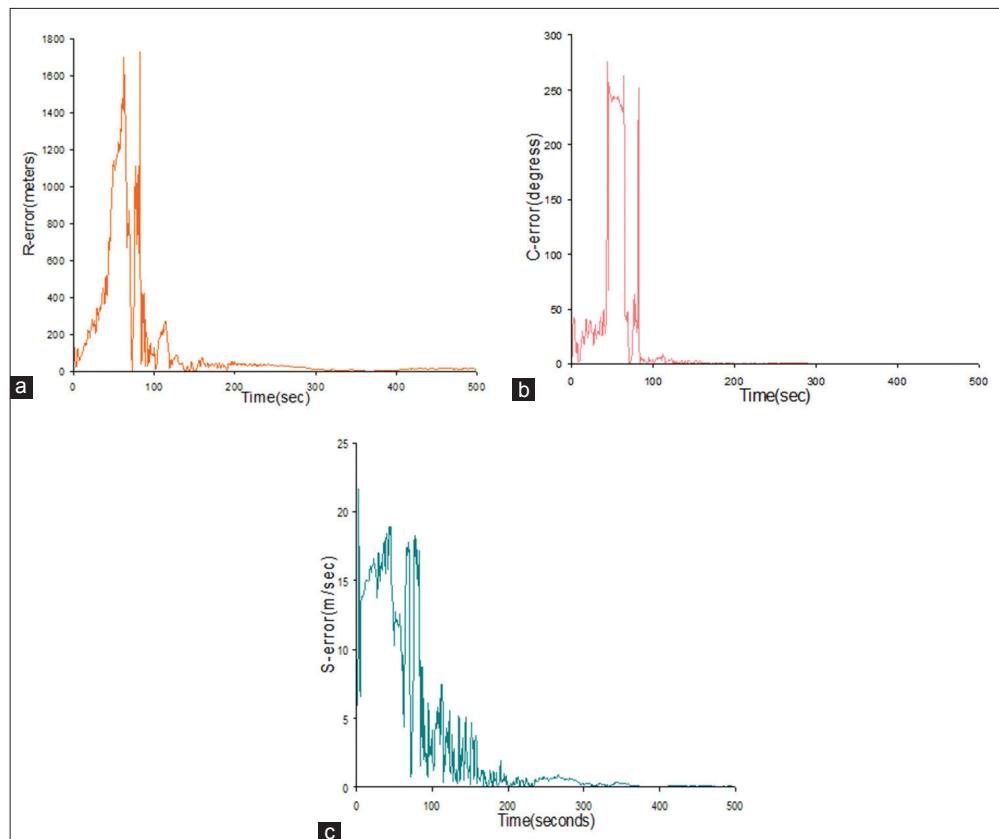


Fig. 1: (a) Error in range estimates. (b) Error in course estimate. (c) Error in speed estimate

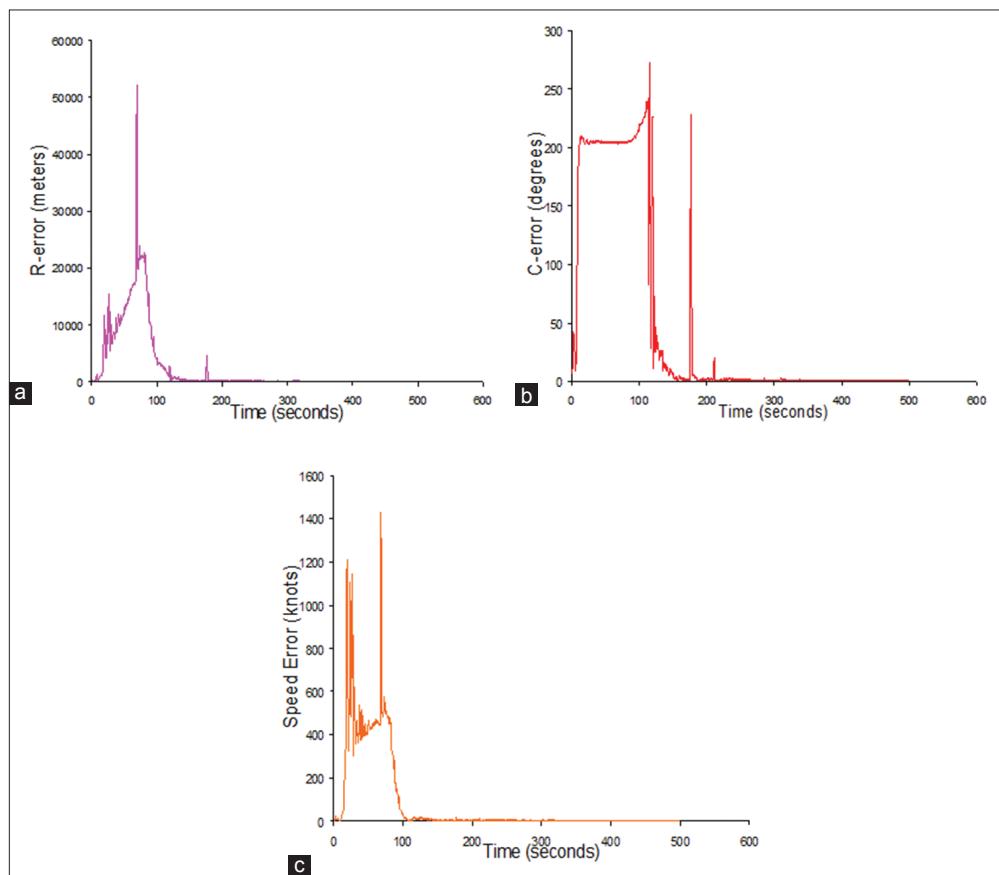


Fig. 2: (a) Error in range estimates. (b) Error in course estimate. (c) Error in speed estimates

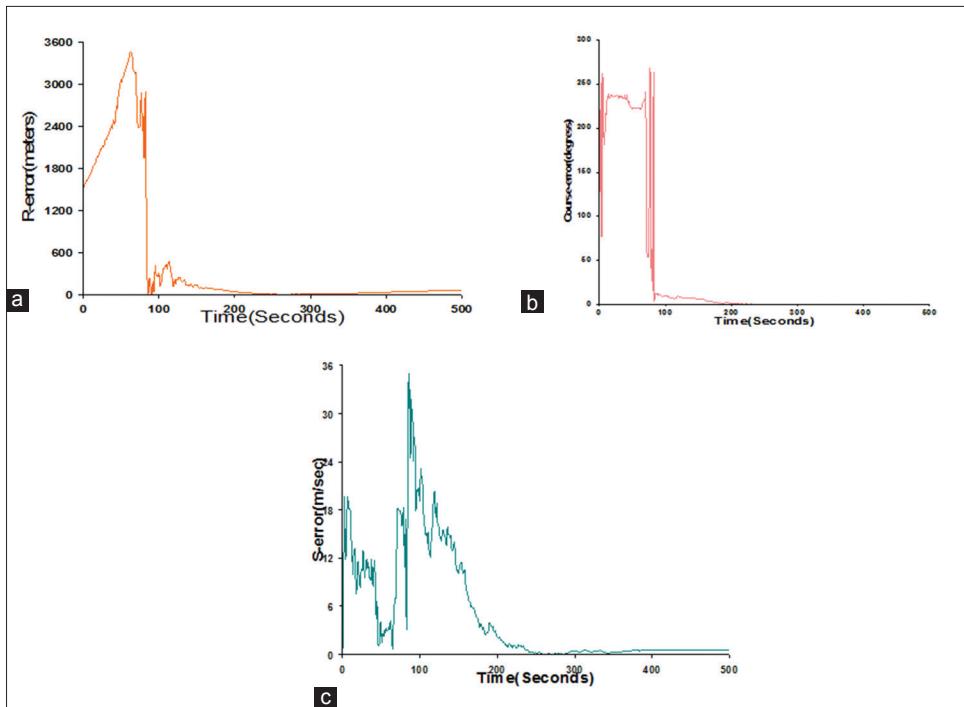


Fig. 3: (a) Error in range estimates. (b) Error in course estimate. (c) Error in speed estimate

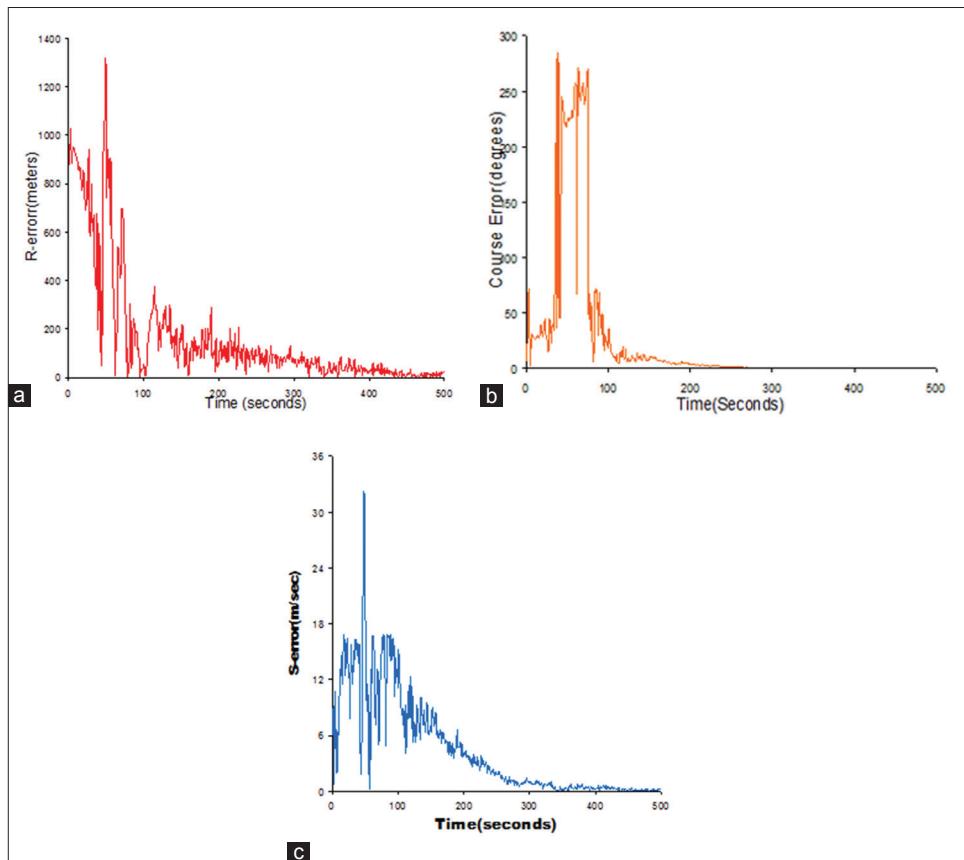


Fig. 4: (a) Error in range estimates. (b) Error in course estimates. (c) Error in speed estimates

are not available continuously. The bearings available in manual mode are highly inconsistent and are not useful for good tracking of the target.[22-26] In this algorithm, it is assumed that good track continuity is maintained over the simulation period. This means that

propagation conditions are satisfactory during this period as well as track continuity is maintained during ownship manoeuvres. The algorithm cannot provide good results when the measurement noise is more than  $1^\circ$  rms.

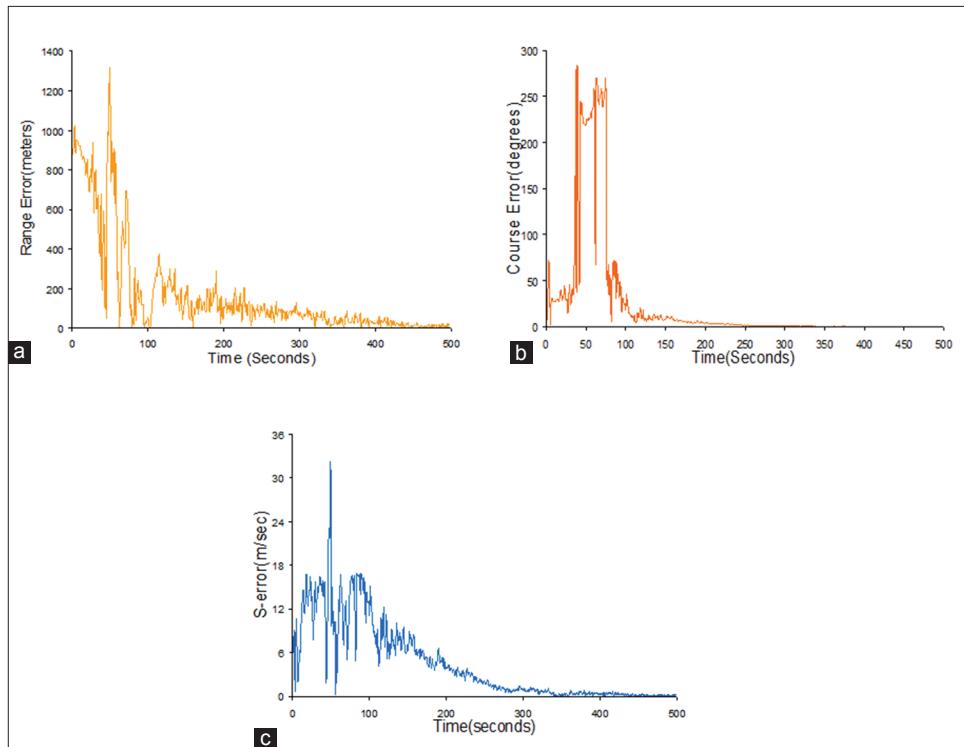


Fig. 5: (a) Error in range estimates. (b) Error in course estimates. (c) Error in speed estimates

Table 2: Convergence of various algorithms

Scenario	Convergence time with one MGBEKF (seconds)			Convergence time with PMGBEKF (seconds)			Convergence time with outliers using PMGBEKF (seconds)		
	Range	Course	Speed	Range	Course	Speed	Range	Course	Speed
Scenario 1	115	161	192	84	124	145	140	183	312
Scenario 2	111	235	260	76	199	205	191	214	407

MGBEKF: Modified gain bearings-only extended Kalman filter, PMGBEKF: Parameterized modified gain bearings-only extended Kalman filter

## CONCLUSION

In this paper, tracking of torpedo using HMA measurements is explored. Ownship safety maneuver is used for observability of the process. Effort is made to reduce the convergence time or to obtain torpedo motion parameters as quickly as possible to carry out evasive maneuver by ownship. The convergence time is reduced with the usage of parameterisation in initialization of target state vector. The target range, course, and speed intervals of interest are divided into a number of sub-intervals, and each subinterval is dealt with an independent MGBEKF. The performance of the PMGBEKF is superior to that of MGBEKF. Extensive simulation is carried out and the results are found to be satisfactory.

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#### APPENDIX A

In safety maneuver algorithm, it is observed whether the torpedo is on port side (sign of the torpedo is negative) or starboard side (sign of the torpedo is positive). If the absolute value of the relative bearing is  $<70^\circ$ , then evasive maneuver is equal to measured bearing+(sign of torpedo side)\* $30^\circ$ . If the absolute value of relative bearing is greater than or equal to  $70^\circ$ , then evasive maneuver is equal to  $180^\circ$ +measured bearing+(sign of torpedo side)\* $30^\circ$ .