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Research Article

ANALYSIS OF EXTENDED KALMAN FILTER USING RANGE AND LINE OF SIGHT MEASUREMENT FOR UNDERSEA TARGET LOCALISATION

JAWAHAR A, VISHNU CHAKRAVARTHI CH

Department of Electrical and Electronics Engineering, Sanketika Institute of Technology and Management, Visakhapatnam, Andhra Pradesh, India. Email: 0891jawahar@gmail.com

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ABSTRACT

Objectives: The feasibility of the extended Kalman filter using range and bearing measurements is explored for underwater applications.

Methods: The input estimation technique, developed by Bar-Shalom and Fortmann for radar applications is implemented for sonar applications. Input estimation is used to estimate the target acceleration whenever the target makes a maneuver. The algorithm estimates target motion parameters and detects target maneuver using zero mean Chi-square distributed random sequence residual.

Results: On detection of target maneuver, this algorithm corrects the velocity and position components using acceleration components.

Conclusion: Finally, the performance of this algorithm is evaluated in Monte-Carlo simulations and results are shown for various typical geometries and found that this input estimation technique can be used for underwater applications.

Keywords: Sonar, Estimation, Simulation, Kalman filter, Maneuver, Bearing, Range.

INTRODUCTION

In the ocean environment, two-dimensional target motion analysis is generally used. A sonar positioned on a ship observes noisy bearing and range measurements of the target in active mode. The observer is assumed to be moving in straight line, and the target is assumed to be moving mostly in straight line with maneuver occasionally. The observer processes the measurements and estimates the target motion parameters, *viz*., range, course, bearing, and speed of the target. As an illustration, the target motion problem with a single moving observer is shown in Fig. 1. Rich literature is available to track a target using range and bearing measurements [1-6]. In this paper, the authors try to apply Kalman filter for the sea scenario using the input estimation technique to take care of target maneuver.

The difference between the measurements and the estimated measurements is termed as innovations. It is observed that the innovation sequence follows Chi-square distribution. From the innovations of the Kalman filter based on the non-maneuvering model, the acceleration input is detected, estimated and the same is used to correct the state estimate. This process is done using a sliding window of the latest "s" (s is a design factor) measurements. During this window period, the input is assumed to be constant. This procedure is called input estimation and is given in detail in reference [7-15]. Input estimation developed by Bar-Shalom and Fortmann is used so far for radar applications, in which measurements are available continuously. Here, effort is made to utilize the technique for underwater sonar applications, in which the measurements are available at discrete intervals.

There are mainly two versions of Kalman filter – A linearized Kalman filter, in which polar measurements are converted into Cartesian coordinates and the well-known extended Kalman filter (EKF). Recently, Pork and Lee [8] presented a detailed theoretical comparative study of the above two methods and stated that both the methods perform well. Here, EKF is used throughout the paper. Section 2 describes the mathematical modeling of measurements.

MATHEMATICAL MODELLING

The target needs to be tracked using noise-corrupted range and bearing measurements. For the purpose of introducing concepts, target is assumed to be moving with constant velocity. The target state equation is given by:

$$X_s(k+1)=\Phi(k+1/k)X_s(k)+b(k)+\tau(k)\omega(k) \tag{1}$$

$X_s(k)$ is the state vector with target velocity and range components and is given by:

$$X_S(k) = \begin{bmatrix} \dot{x}(k) & \dot{y}(k) & Rx(k) & Ry(k) \end{bmatrix}^T \tag{2}$$

$\Phi(k+1/k)$ is the state transition matrix and is given by:

$$\phi(k+1/k) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ t & 0 & 1 & 0 \\ 0 & t & 0 & 1 \end{bmatrix} \tag{3}$$

Where, t is the sample time,

$$b(k) = \begin{bmatrix} 0 & 0 & -(x_0(k+1)-x_0(k)) & -(y_0(k+1)-y_0(k)) \end{bmatrix}^T \tag{4}$$

Where x_0 and y_0 are observer position components, respectively. True North convention is followed for all angles to reduce mathematical complexity and easy implementation.

$$\text{Where } \tau(k) = \begin{bmatrix} t & 0 \\ 0 & t \\ \frac{t^2}{2} & 0 \\ 0 & \frac{t^2}{2} \end{bmatrix} \tag{5}$$

$\omega(k)$ is plant noise with covariance $Q(k)$,assumed to be Gaussian and uncorrelated with measurement noises. The measurement vector $Z(k)$ is given by:

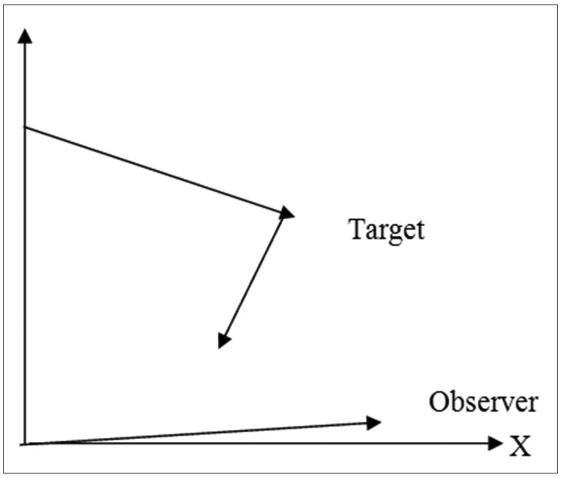

Fig. 1: Typical target observer encounter

$$Z(k) = \begin{bmatrix} B_m(k) & R_m(k) \end{bmatrix}^T \tag{6}$$

Where $R_m(k)$ and $B_m(k)$ are range and bearing measurements and are given by:

$$R_m(k)=R(k)+\xi_R(k) \tag{7}$$

$$B_m(k)=B(k)+\xi_B(k) \tag{8}$$

Where, $R(k)$ and $B(k)$ are actual range and bearing. ξ_R and ξ_B are Gaussian noises with $\sigma_R^2 = E[\xi_R^2]$, $\sigma_B^2 = E[\xi_B^2]$ and $E[\xi_R \xi_B]=0$.

The measurement matrix $H(k)$ can be shown as:

$$\text{H(k)=}\ H(k) = \begin{bmatrix} 0 & 0 & \dfrac{\cos(B)}{R} & \dfrac{-\sin(B)}{R} \\ 0 & 0 & \sin(B) & \cos(B) \end{bmatrix} \tag{9}$$

Since the true bearing and range are not available in practice, these are replaced by the estimated bearing and range. The five equations of the Kalman filter are given by:

Prediction:

State: $X(k+1/k)=\Phi(k+1/k)X(k/k)+b(k+1)$ (10)

Covariance: $P(k+1/k)=\Phi^T(k+1/k)P(k/k)\Phi(k+1/k)+Q(k+1)$ (11)

Kalman gain:

$$G(k+1)=P(k+1/k)H^T(k+1)\left[H(k+1)P(k+1/k)H^T(k+1)+R(k+1)\right]^{-1} \tag{12}$$

Correction:

State: $X(k+1/k+1)=X(k+1/k)+G(k+1)\left[Z(k+1)-\hat{Z}(k+1)\right]$ (13)

Covariance: $P(k+1/k+1)=\left[I-G(k+1)H(k+1)\right]P(k+1/k)$ (14)

Where, $\hat{Z}(k+1)$ is estimated measurement. The target motion parameters - range, bearing, course, and speed (R, B, C, S) are calculated from the estimated state vector as follows.

$$R(k)=\sqrt{R_x^2(k)+R_y^2(k)^2} \tag{15}$$

$$B(k)=\mathrm{Tan}^{-1}\left(\frac{R_x(k)}{R_y(k)}\right) \tag{16}$$

$$C(k)=\mathrm{Tan}^{-1}\left(\frac{\dot{x}(k)}{\dot{y}(k)}\right) \tag{17}$$

$$S(k)=\sqrt{\dot{x}(k)^2+\dot{y}(k)^2} \tag{18}$$

INPUT ESTIMATION FOR SONAR APPLICATIONS

Consider the system with state equation:

$$X(k+1)=\Phi(k+1/k)X(k)+\mathrm{F}u(k)+\tau(k)\omega(k) \tag{19}$$

Where, u is an unknown input modeling the target maneuvers and the F matrix is given by

$$F=\begin{bmatrix} t & 0 \\ 0 & t \\ \dfrac{t^2}{2} & 0 \\ 0 & \dfrac{t^2}{2} \end{bmatrix} \tag{20}$$

Here, the final equations are produced. Assume that the target starts maneuvering at time k. It's unknown inputs during the time interval [k,....,$k+s$] are $u[i]$, $i=k,\ldots,k+s-1$. Detailed derivation is available in reference [3].

The innovations corresponding to the correct filter are given by:

$$v(k+1)=z(k+1)-HX(k+1/k) \tag{21}$$

And the innovations for the non-maneuvering model (now mismatched model) are:

$$v^*(k+1)=z(k+1)-H\hat{X}^*(k+1/k) \tag{22}$$

$v^*(k+1)$ can be written as:

$$v^*(k+1)=\Psi(i+1)u+v(k+1),\ i=k,\ldots k+s-1 \tag{23}$$

Where,

$$\Psi(i+1)=H\sum_{j=k}^{i}\left[\prod_{m=j+1}^{i}\varphi(m)\right]F \tag{24}$$

Where,

$$\varphi(i)=\Phi[I-G(i)H] \tag{25}$$

Equation (25) shows that the innovation γ^* of the non-maneuvering filter is a linear measurement of the input (maneuver). u is the presence of the additive "white noise," γ. From Equation (24), it follows that the input can be estimated using the least square criterion and the estimation can be done in batch form as:

$$\hat{u}=(\Psi^T S^{-1}\Psi)^{-1}\Psi^T S^{-1}y \tag{26}$$

Where, S is a covariance matrix of $\gamma(k)$ and it is given by S=diag [$S(i)$] (27)

The covariance matrix of $\hat{u}$ is given by:

$$L=\left(\Psi^T S^{-1}\Psi\right)^{-1} \tag{28}$$

A maneuver is declared detected, only if it is statistically significant. The significance test for the vector estimate $\hat{u}$ is:

$$d(\hat{u}) = \hat{u}^T L^{-1} \hat{u} \geq c \tag{29}$$

Where, c is a threshold. As the estimate $\hat{u}$ is a normal random variable with mean zero and covariance L, then the statistic d is Chi-square distributed with n degrees of freedom and c is chosen such that the probability of false alarm is,

$$P\left\{ d(\hat{u}) \geq c \right\} = \alpha \text{ with } \alpha = 10^{-2} \text{ or smaller} \tag{30}$$

If a maneuver is detected, then the state has to be corrected as follows.

$$\hat{a}^{u}(k+s+1/k+s) = \hat{x}^{*}(k+s+1/k+s) + \mathrm{Mu} \tag{31}$$

$$\text{Where } M = \sum_{j=k}^{k+s} \left[\prod_{m=j+1}^{k+s} \Phi(m) \right] F \tag{32}$$

The covariance associated with the estimate is:

$$P^{u}(k+s+1/k+s) = P(k+s+1/k+s) + MLM^{T} \tag{33}$$

A maneuver is considered finished when the input estimate based on measurements from the sliding window of length s becomes insignificant. The length s is a design parameter.

SIMULATION AND RESULTS

The algorithm is realized using Matlab. For the implementation of the algorithm, the initial estimate of the target state vector is chosen as follows.

$$X(0/0) = \begin{bmatrix} 10 & 15 & R_m \sin(B_m) & R_m \cos(B_m) \end{bmatrix}^T \tag{34}$$

Here, the velocity components are assumed to be 10 and 15 m/s, which are close to the realistic speed of the vehicles on seawater. The initial covariance matrix P (0/0) can be comfortably taken as unit diagonal matrix. The observer is assumed to be moving in straight line, at a constant speed of 18 m/s at a course of 60°. The underwater target is assumed to be moving at a speed of 6 m/s and at an initial range of 20,000 m with the initial bearing 60° relative to the observer. The noise in the bearing and Range are assumed to be 0.33° and 7 m r.m.s., respectively. The plant noise is chosen as 0.01. The time interval between the measurements is initially around 27 seconds. It reduces as subsequently the range gets decreased. Here, the transient matrix is not considered as a constant and it is updated along with the Kalman filter equations, whenever the measurements are obtained. The simulation is carried out for 30 minutes.

The number of scenarios is tested by changing the course of the target in steps of 10° in such a way that the angle between the target course and LOS is always <55°, as the closing targets are of interest to the observer. In general, the errors allowed in the estimated target motion parameters are 8% in the range, 0.2° in the bearing, 6° in the course, and 3 m/s in velocity estimates. The results of these scenarios in Monte-Carlo simulations are noted and for the purpose of analysis a scenario at a target course equal to 140° as shown in Fig. 2 is considered. In subsequent Fig. 2a-d, the errors in the range, course, bearing and speed estimates are denoted by RngError, CrsError, BrgError and SpdError, respectively. The range, course, bearing and speed are converged at 4th sample (105 seconds), 13th sample (310 seconds), 4th sample (105 seconds), and 6th sample (154 seconds), respectively. From the results, it is observed that the total solution with the required accuracy is obtained from the 13th sample (310 seconds) onward.

In general, the window size of at least five samples is taken so that the reliability is increased in the highly noisy environment prevalent in underwater scenarios. The higher the window size the higher is the value of $d(\xi)$. This window size is determined on the basis of the results of Monte-Carlo simulations against the number of geometries. If the window size is <5, it drastically reduces the reliability. Hence, a 5-sample window is employed.

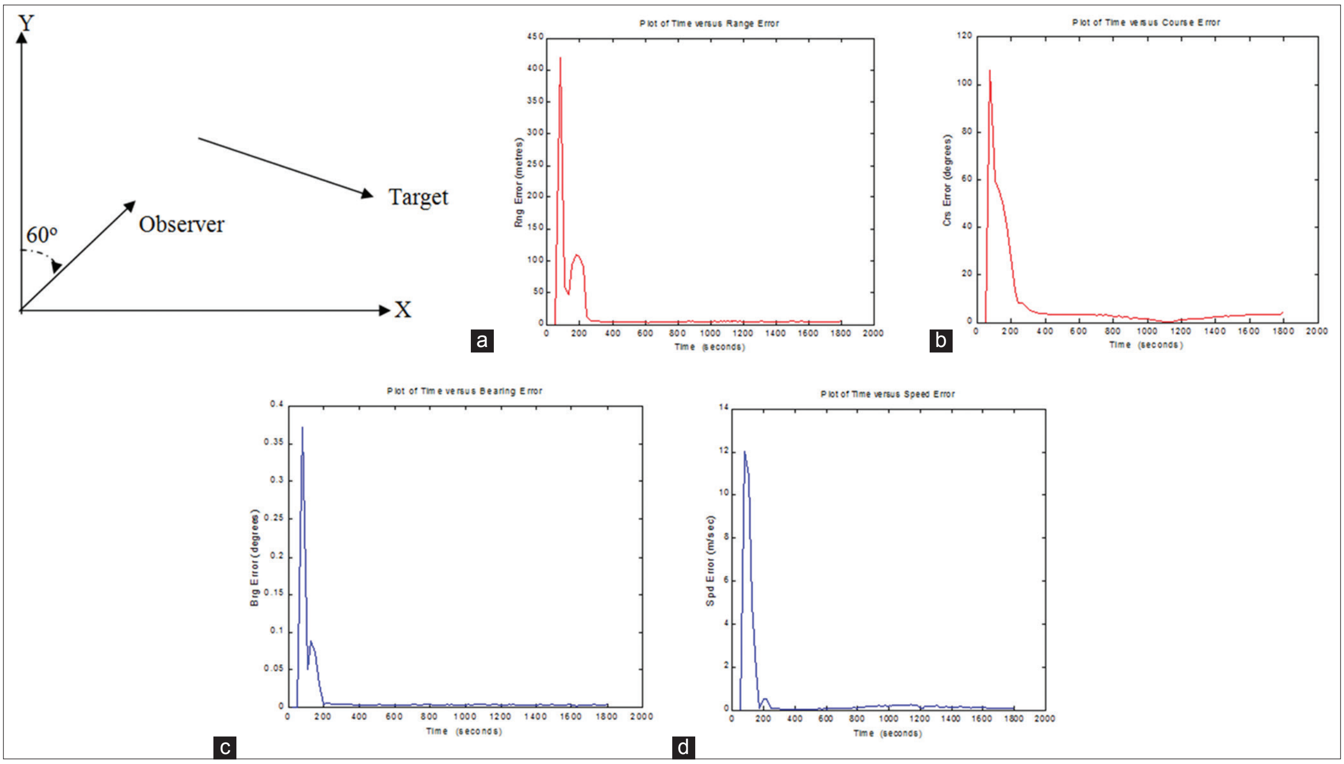

Fig. 2: Target motion analysis with single observation platform with straight run target, (a) Error in range estimates, (b) error is course estimates, (c) error in bearing estimates, (d) error in speed estimates

The theoretical value of the Chi-square variable with 5 degrees of freedom at 90% confidence level is 9.24, and the same value is considered for maneuver detection. The scenario is run 100 times in Monte-Carlo Simulation and it is observed that the solution is obtained around 14th sample (330 seconds). Let us say that by 330 seconds, the process is stabilized. The statistic at this time is around 0.1. Thereafter, it never increased to more than 0.4.

The geometry shown in Fig. 3 is extended as follows. The target is assumed to do course maneuver at the time of 600 seconds from 140° to 45° with a turning rate of 3°/seconds as shown. The results after 100 Monte-Carlo runs are shown in Fig. 3a-d.

The maneuver is given between 29th sample (591 seconds) and 30th sample (605 seconds). The maneuver is continued and completed

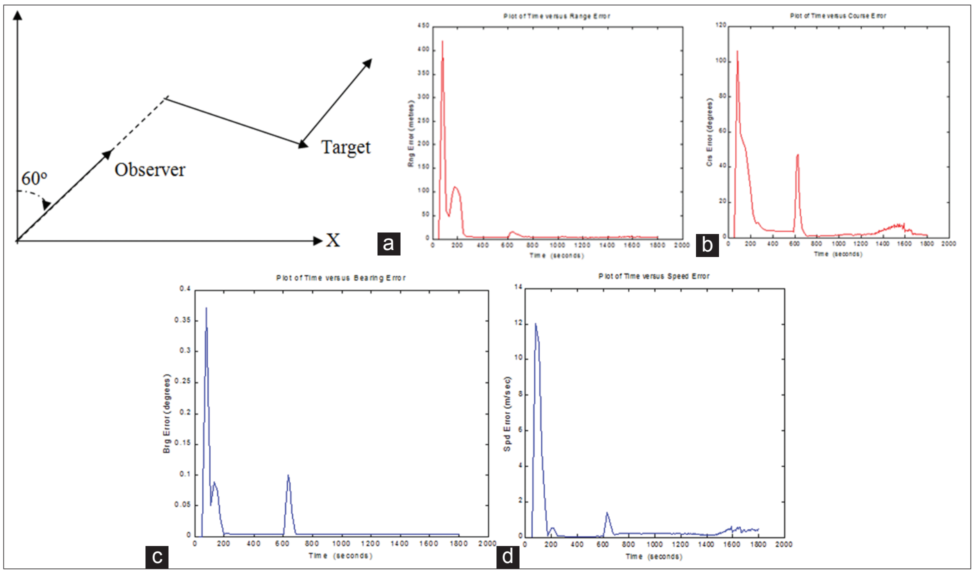

Fig. 3: Target motion analysis with single observation platform with target maneuver, with target maneuver (a) Error in range estimates, (b) error in course estimates, (c) error in bearing estimates, (d) error in speed estimates

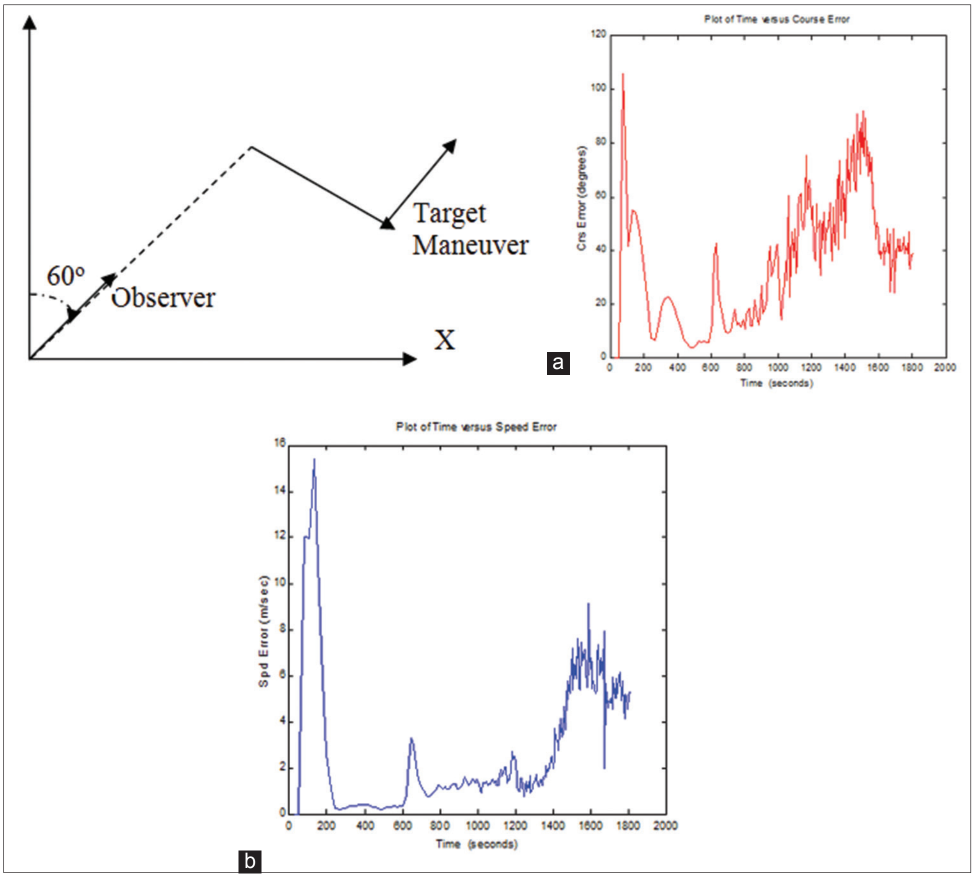

Fig. 4: Target motion analysis with single observation platform, (a) Error in course estimates, (b) error in speed estimates

Table 1: Convergence time

Sample number	Time	Statistic
30	605	1.1
31	620	25.9
32	635	252
33	649	534
34	662	639
35	676	522
36	690	262
37	704	64

between 31st (620 seconds) and 32nd sample (635 seconds). The change in statistic at various timings is shown in Table 1.

The maneuver is detected first at the 31st sample and continued to show up to 37th sample (704 seconds). Actually, the target maneuver is completed by 32nd sample but the statistic is reduced to below 9.4 after 37th sample (704 seconds). Thereafter, the statistic is reduced to around 0.4 and the process is stabilized.

The range estimate is not disturbed due to maneuver. The course and bearing speed are converged at 34th sample (662 seconds) and speed at 32nd sample (635 seconds). It is observed that the total solution with the required accuracy is obtained from the 34th sample (662 seconds) onwards.

More or less the practical requirement specifications are matching with that of theoretical threshold.

LIMITATIONS OF FILTER

The filter cannot provide good results when the measurements are corrupted with heavy noise. For the purpose of illustration, in the scenario Fig. 4, the noise in the range measurement is assumed to be 100 m r.m.s instead of 7 m r.m.s. The results after 100 Monte-Carlo runs are evaluated, and the errors in course and speed are produced in Fig. 4a and b. It is clear from the figures that the filter is not able to its job when the noise in measurements is more.

REFERENCES

AQ1

8. Pork ST, Lee JG. Design of a practical tracking algorithm with radar measurements. IEEE Trans Aerosp Electron Syst 1998;34(4):1337-44.
13. Bar-Shalom Y, Fortman TE. Tracking and Data Association. San Diego, USA: Academic Press Inc.; 1991.
15. Bar-Shalom Y, Lee XR. Estimation and Tracking Principles, Techniques and Software. Boston, London: Artech house; 1993.

Author Query???

AQ1: Kindly provide references 1-7, 9-12 and 14 in reference list