INNOVARE JOURNAL OF SCIENCES

Vol 8, Special Issue 1, 2020

INNOVARE ACADEMIC SCIENCES
Knowledge to Innovation

ISSN-2321-5496

Full Proceeding Paper

FRACTIONAL CALCULUS, SHORT HISTORY

SYEDA FARRA FATIMA NAHRI,

Department of Mathematics, Milliya Arts, Science and Management Science College, Beed, India. Email :syedafarra@gmail.com

Received: 25 January 2020, Revised and Accepted: 17 March 2020

ABSTRACT

We know how to find derivatives .But it is a question that how to find derivatives if the order of differentiation to be negative or non integer. To find such types of derivatives, field of mathematics that is fractional calculus is used. Fractional order derivatives and integrals give a new dimension to mathematicson one hand and many problems in physical sciences can be expressed and solved successfully by using fractional calculus on the other hand. In this paper we discuss in brief fractional calculus.

Keywords: Fractional calculus, Derivative, Integration, Definite integral

INTRODUCTION

The idea of differentiation and integration is not new for non integer. Leibniz , 30th September 1695 wrote a letter toL'Hospital and mentioned regarding it[1]. In this letter he created a mention of $d^{\frac{1}{2}}x \ and \ d^{\frac{1}{2}}y$ and called it half derivative. However,very little additional systematic study of fractional calculus seems to have been done in the beginning and middle of nineteenth century by Liouville, Riemann and Holmgren, although Euler and Lagrange also made some contribution.Liouville expanded functions in series of exponentials and outlined the qth derivative of such series by operating term by term for positive integer q. Riemann gives another definition including a definite integral and was applicable to power series with non integer exponent. In the twenteeth century, the main contributors to fractional calculus are Weyl(1917), Hardy (1917), Littlewood (1925-1932), Kober (1940), Kuttner (1953), Erdelyi (1964) has applied fractional calculus to integral equations, while Higgins (1967) used fractional integral operators to solve some differential equations. Other applications include application to Electrochemistry by Belavin(1964), Oldham 1969, Spanier 1970, Grenness (1972) and to general transport problems (Oldham,1973, Spanier 1972) and also diffusion problem (Oldham and Spanier).

It is quite common to encounter differential operators $\frac{d}{dx}, \frac{d^2}{dx^2}, \frac{d^3}{dx^3}, etc$ in calculus. But there not be operators like $\frac{d^{\frac{1}{2}}}{dx^{\frac{1}{2}}}, \frac{d^{-1}}{dx^{-1}}, \frac{d^{\sqrt{2}}}{dx^{\sqrt{2}}}$ $etc.$We observe that operator $\frac{d^{-1}}{dx^{-1}}$ is nothing but indefinite integral in disguise. However fractional orders of differentiationare more mysterious, as they have no obvious geometric interpretation in terms of slope (for differentiation) and area for integration. However, Fractional order derivatives and integrals provides a new dimension to mathematics on one hand and many problems in physical sciences are expressed and resolved with success by using fractional calculus on the other hand. In this paper we tend to discuss in short fractional calculus. Understanding of definitions and use of fractional calculus will be made more clear by simple mathematical definitions like Gamma function, Beta function, The Laplace transform etc , that will arise within the study of it.

Some definitions of Fractional derivatives:

The Laplace definition of a fractional derivative of a signal x(t) is [2]

$$D^{\propto}x(t) = L^{-1}\{s^{\propto}X(s) - \sum_{k=0}^{n-1}\binom{n}{k}s^{k}D^{\propto-k-1}x(t)/t=0\}$$

where $n-1 < \propto \leq n, \ \propto > 0$. The Grunwald-Letnikov definition is given by $(\propto \in R)$: [2]

$$D^{\propto}x(t) = \lim_{h\to 0}[\frac{1}{h^{\propto}}\sum_{k=0}^{\infty}(-1)^{k}\binom{\propto}{k}x(t-kh)]$$

$$,\binom{\propto}{k} = \frac{\Gamma(\propto+1)}{\Gamma(k+1)\Gamma(\propto-k+1)}$$

whereΓ is that the Gamma operator and h is that the time increment. The Riemann- Liouville definition of the fractional- order derivative is $(\propto > 0)$; [2]

$$D_t^{\propto}f(t) = \frac{1}{\Gamma(n-\propto)}\frac{d^n}{dt^n}\int_a^t \frac{f(\tau)}{(t-\tau)^{\propto-n+1}}d\tau, \ \ n-1<\propto<n$$

Basedon these definitionsit is possible to calculate the fractional-order integrals/ derivatives of several functions. In recent years Fractional calculus has been a fruitful field of research in science and engineering. FC is used in the field of heat diffusion, redundant robots and digital circuit synthesis, Viscoelasticity and damping, system identification , genetic algorithms and also economics and finance.

Some definitions of Fractional integration: Riemann- Liouville left –sided integral:[5]

$$I_{a^+}^{\propto}[f(x)] = \frac{1}{\Gamma(\propto)}\int_a^x (x-\xi)^{\propto-1}f(\xi)d\xi \ ,x \geq a$$

Riemann- Liouville right –sided integral:[5]

$$I_{b^-}^{a}[f(x)] = \frac{1}{\Gamma(\propto)}\int_x^b (\xi-x)^{\propto-1}f(\xi)d\xi \ ,x \leq b$$

Riemann- Liouville definitions of calculus are used mostly .The definition given byGrunwald-Letnikov used in signal processing, engineeringand control. Fractional derivatives and integrations are applicable in Classical calculus. [3]

CONCLUSION

Fractional Calculus has been a fruitful field of research in science and engineering and many other areas too. This mathematical tool will be used in development of new application of Fractional calculus in research and applications. The study of it opens the new branches of thoughts.

REFERENCES

1. http://pdfs.semanticscholar.org^...PDFFractional Calculus: History, Definitions and Applications for the Engineering
2. www.hindawl.com Web results Some Applications of Fractional Calculus in Engineering
3. www.hindawl.com A Note on Fractional Order Derivatives and Table of Fractional Derivatives of…
4. https://www.hindawi.com/journals/mpe/2014/238459/ A Review of Definitions for Fractional Derivatives and Integral