INNOVARE JOURNAL OF SCIENCES

Vol 8, Special Issue 1, 2020 ISSN-2321-5496

Full Proceeding Paper

A STUDY OF WAVE EQUATION BY SEPARATION OF VARIABLES

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Received: 25 January 2020, Revised and Accepted: 17 March 2020

ABSTRACT

Objective: In this paper we focus our study on wave equations. Westudying the solution of wave function by using separation of variables technique. The functions of several variablesand having worked through the concept of a partial derivative.

Materials and Methods: We first formulate the wave function u(x,t) where x is length of string. Solving the equation$u(x,t) = F(x)G(t)$ in two variables byusing the methods of Partial Differential Equation. We get the following equation

$$\frac{G''(t)}{c^2G(t)} = \frac{F''(x)}{F(x)} = k$$

where k is constant.

Results: We are going to check the possible for the constant k in the above equation. First we consider k = 0 then we get u(x, t)= (px+r) (at+b) where a,b, p and r, were constants.

Secondly we consider k> 0 then we get $F(x) = Ae^{\omega x} + Be^{-\omega x}$ whereA and B are constants and $\omega = \sqrt{k}$.

Lastly we consider k< 0 then we get $u_n(x,t) = F(x)G(t) = \left(C\cos(\lambda_n t) + D\sin(\lambda_n t)\right)\sin\left(\frac{n\pi}{l}x\right)$ where the integer n that was used isidentified by the subscript in $u_n(x,t)$ and λ_n , and arbitrary constants are C and D.

Conclusion: The solutions given in the first two cases are dull solutions.The solution given in the last case really does satisfy the wave equation. We can find a particular solution function for varying values of time, t,

Keywords: - Wave equation, Partial derivatives, Exponential functions, Frequency, Poisson equation.

INTRODUCTION

We consider a close look at the PDEs. Try to classify using the given terminology. Note that the f(x,y) function in the Poisson equation is just a function of the variables x and y, it has nothing to do with u(x,y). To solve Partial Differential Equations is considerably more difficult in general than to solve Ordinary Differential Equations, as the complications involved can be great.The wave equations can be solved by several approaches. The first one will using a technique called separation of variables.[1] The second technique, used is a transformation trick that also reduces the complexity of the original PDE, but in a very different manner.

The advantage of an abstract approach is that it concentrates on the required facts, so that these facts become clearly visible and one's attention is not disturbed by non important details. Moreover, by developing a box of tools in the abstract framework, one is equipped for solving many different problems. In the abstract approach, we can usually starts from a set of elements satisfying certain axioms. The theory then consists of logical consequences which are derived from the axioms and are derived as theorems once and for all. These general theorems can then later be applied to various concrete special sets satisfying these axioms.

We will develop such an abstract scheme for doing calculus in function spaces and other infinite-dimensional spaces, and this is what this course is about.[2]We will be equipped with a set of tools for solving these problems, and in particular, we will return to the optimal mining operation problem again and solve it.

MATERIALS AND METHODS

First, note that for a particular wave equation situation, in addition to the Partial Differential Equation, we will also consider boundary conditions arising from the fact that the endpoints of the string are attached solidly, x = 0 at the left end of the string.At the other end of the string, we suppose has overall length l. Let's start the process of solving the Partial Differential Equation by first figuring out what these boundary conditions imply for the solution function i.e. $u(x,t)$.

(1) $u(0,t) = 0$ and $u(l,t) = 0$

for all values of t are the boundary conditions for this wave equation [5]. These will be key when we later on need to sort through probable solution functions for functions that will satisfy our particular vibrating string set-up.

Note that we probably need to specify what the shape of the string is right when time t = 0, and you are right to come up with a particular solution function, we wantto know $u(x,0)$. In fact we will also need to know the initial velocity of the stringi. e. $u_t(x,0)$.

These two requirements are called the initial conditions for this wave equation, and are also necessary to specify a specific vibrating string solution. For instance, as the simplest example of initial conditions, if no one is plucking the string, and it's perfectly flat to start with, then the initial conditions will be $u(x,0) = 0$ (for perfectly flat string) with initial velocity, $u_t(x,0) = 0$ [5,6] Here, the solution function is pretty unenlightening, it's just $u(x,t) = 0$ when no movement of the string through time.

To use the separation of variables technique we make the key assumption that whatever the solution function is, that it can be written as the multiplication of two independent functions, each one